13

Universal Gravitation

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q13.1** The force is proportional to the product of the masses and inversely proportional to the square of the separation distance, so we compute m_1m_2/r^2 for each case:
(a) $2\cdot 3/1^2 = 6$ (b) 18 (c) $18/4 = 4.5$ (d) 4.5 (e) $16/4 = 4$. The ranking is then b > a > c = d > e.

***Q13.2** Answer (d). The International Space Station orbits just above the atmosphere, only a few hundred kilometers above the ground. This distance is small compared to the radius of the Earth, so the gravitational force on the astronaut is only slightly less than on the ground. We think of it as having a very different effect than it does on the ground, just because the normal force on the orbiting astronaut is zero.

***Q13.3** Answer (b). Switching off gravity would let the atmosphere evaporate away, but switching off the atmosphere has no effect on the planet's gravitational field.

Q13.4 To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

***Q13.5** Having twice the mass would make the surface gravitational field two times larger. But the inverse square law says that having twice the radius would make the surface acceleration due to gravitation four times smaller. Altogether, g at the surface of B becomes $(2\text{ m/s}^2)(2)/4 = 1\text{ m/s}^2$, answer (e).

***Q13.6** (i) $4^2 = 16$ times smaller: Answer (i), according to the inverse square law.

(ii) $mv^2/r = GMm/r^2$ predicts that v is proportional to $(1/r)^{1/2}$, so it becomes $(1/4)^{1/2} = 1/2$ as large: Answer (f).

(iii) $(4^3)^{1/2} = 8$ times larger: Answer (b), according to Kepler's third law.

***Q13.7** Answer (b). The Earth is farthest from the sun around July 4 every year, when it is summer in the northern hemisphere and winter in the southern hemisphere. As described by Kepler's second law, this is when the planet is moving slowest in its orbit. Thus it takes more time for the planet to plod around the 180° span containing the minimum-speed point.

Q13.8 Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

***Q13.9** Answer (c). Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}$$

Q13.10 The escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to v^2, or 25 times more fuel is required to leave the Earth versus leaving the Moon.

***Q13.11** The gravitational potential energy of the Earth-Sun system is negative and twice as large in magnitude as the kinetic energy of the Earth relative to the Sun. Then the total energy is negative and equal in absolute value to the kinetic energy. The ranking is a > b = c.

Q13.12 For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

Q13.13 Every point q on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point q' for which the components of the gravitational force perpendicular to the axis will cancel. Point q' can be found by rotating the sphere through 180° about the axis. The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the non-uniform sphere still lies along the axis.

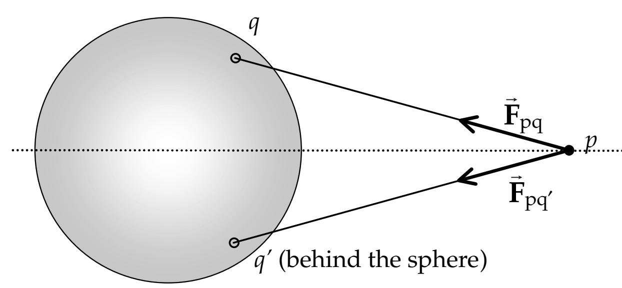

FIG. Q13.13

Q13.14 Speed is maximum at closest approach. Speed is minimum at farthest distance. These two points, perihelion and aphelion respectively, are 180° apart, at opposite ends of the major axis of the orbit.

Q13.15 Set the universal description of the gravitational force, $F_g = \frac{GM_X m}{R_X^2}$, equal to the local description, $F_g = ma_{\text{gravitational}}$, where M_X and R_X are the mass and radius of planet X, respectively, and m is the mass of a "test particle." Divide both sides by m.

Q13.16 The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 13.1 would suggest. All the bits of matter that make up the Earth will pull in different outward directions on the extra particle.

Q13.17 Cavendish determined G. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth.

***Q13.18** The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. But Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet's gravity does net positive work on it. This is a collision because the spacecraft and planet exert forces on each other while they are isolated from outside forces. It is an elastic collision. The planet loses kinetic energy as the spacecraft gains it.

SOLUTIONS TO PROBLEMS

Section 13.1 **Newton's Law of Universal Gravitation**

P13.1 For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(70\ \text{kg})(70\ \text{kg})}{(2\ \text{m})^2}\boxed{\sim10^{-7}\ \text{N}}$$

P13.2 $F = m_1 g = \dfrac{Gm_1m_2}{r^2}$

$$g = \frac{Gm_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00\times10^4\times10^3\ \text{kg}\right)}{(100\ \text{m})^2} = \boxed{2.67\times10^{-7}\ \text{m/s}^2}$$

P13.3 (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed, and from

$$F_g = \frac{Gm_1m_2}{r^2}$$

we have $\sum F = \dfrac{G(50.0\ \text{kg})(500\ \text{kg}-200\ \text{kg})}{(0.200\ \text{m})^2} = \boxed{2.50\times10^{-5}\ \text{N}}$ toward the 500-kg object.

(b) At a point between the two objects at a distance d from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m}-d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2}$$

To solve, cross-multiply to clear of fractions and take the square root of both sides. The result is $d = \boxed{0.245\ \text{m from the 500-kg object toward the smaller object}}$.

P13.4 $m_1 + m_2 = 5.00\ \text{kg}$ $\qquad m_2 = 5.00\ \text{kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00\times10^{-8}\ \text{N} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{m_1(5.00\ \text{kg}-m_1)}{(0.200\ \text{m})^2}$$

$$(5.00\ \text{kg})m_1 - m_1^2 = \frac{\left(1.00\times10^{-8}\ \text{N}\right)\left(0.040\,0\ \text{m}^2\right)}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 6.00\ \text{kg}^2$$

Thus,

$$m_1^2 - (5.00\ \text{kg})m_1 + 6.00\ \text{kg} = 0$$

or

$$(m_1 - 3.00\ \text{kg})(m_1 - 2.00\ \text{kg}) = 0$$

giving $\boxed{m_1 = 3.00\ \text{kg, so } m_2 = 2.00\ \text{kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

P13.5 The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\vec{\mathbf{F}}_{24} = G\frac{m_4 m_2}{r_{24}^2}\hat{\mathbf{j}}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\ \text{kg})(2.00\ \text{kg})}{(3.00\ \text{m})^2}\hat{\mathbf{j}}$$

$$= 5.93\times10^{-11}\,\hat{\mathbf{j}}\text{N}$$

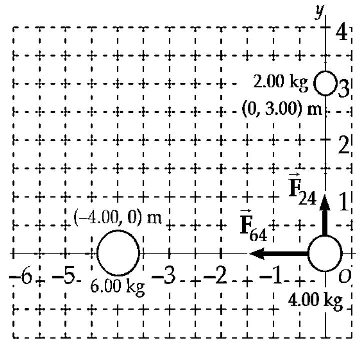

FIG. P13.5

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

$$\vec{\mathbf{F}}_{64} = G\frac{m_4 m_6}{r_{64}^2}\left(-\hat{\mathbf{i}}\right) = \left(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\ \text{kg})(6.00\ \text{kg})}{(4.00\ \text{m})^2}\hat{\mathbf{i}}$$

$$= -10.0\times10^{-11}\,\hat{\mathbf{i}}\text{N}$$

Therefore, the resultant force on the 4.00-kg mass is $\vec{\mathbf{F}}_4 = \vec{\mathbf{F}}_{24} + \vec{\mathbf{F}}_{64} = \boxed{\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right)\times10^{-11}\ \text{N}}$.

***P13.6** (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496\times10^{11}\ \text{m} - 3.84\times10^{8}\ \text{m} = 1.492\times10^{11}\ \text{m}$$

The mass of the Sun, Earth, and Moon are $M_S = 1.99\times10^{30}$ kg

$M_E = 5.98\times10^{24}$ kg

and $M_M = 7.36\times10^{22}$ kg

We have $F_{SM} = \dfrac{Gm_1m_2}{r^2} = \dfrac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\left(7.36\times10^{22}\right)}{\left(1.492\times10^{11}\right)^2} = \boxed{4.39\times10^{20}\ \text{N}}$

(b) $$F_{EM} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\right)\left(7.36\times10^{22}\right)}{\left(3.84\times10^{8}\right)^2} = \boxed{1.99\times10^{20}\ \text{N}}$$

(c) $$F_{SE} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\right)\left(5.98\times10^{24}\right)}{\left(1.496\times10^{11}\right)^2} = \boxed{3.55\times10^{22}\ \text{N}}$$

(d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.7 $$F = \frac{GMm}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(1.50\ \text{kg})\left(15.0\times10^{-3}\ \text{kg}\right)}{\left(4.50\times10^{-2}\ \text{m}\right)^2} = \boxed{7.41\times10^{-10}\ \text{N}}$$

P13.8 Let θ represent the angle each cable makes with the vertical, L the cable length, x the distance each ball scrunches in, and $d = 1$ m the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have

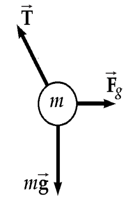

FIG. P13.8

$$\sum F_y = 0: \qquad T\cos\theta - mg = 0$$

$$\sum F_x = 0: \qquad T\sin\theta - \frac{Gmm}{r^2} = 0$$

Then

$$\tan\theta = \frac{Gmm}{r^2 mg} \qquad \frac{x}{\sqrt{L^2 - x^2}} = \frac{Gm}{g(d-2x)^2} \qquad x(d-2x)^2 = \frac{Gm}{g}\sqrt{L^2 - x^2}$$

The factor $\frac{Gm}{g}$ is numerically small. There are two possibilities: either x is small or else $d - 2x$ is small.

Possibility one: We can ignore x in comparison to d and L, obtaining

$$x(1\text{ m})^2 = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}{(9.8\ \text{m/s}^2)}45\text{ m} \qquad x = 3.06\times10^{-8}\text{ m}$$

The separation distance is $r = 1\text{ m} - 2(3.06\times10^{-8}\text{ m}) = \boxed{1.000\text{ m} - 61.3\text{ nm}}$. This equilibrium is stable.

Possibility two: If $d - 2x$ is small, $x \approx 0.5$ m and the equation becomes

$$(0.5\text{ m})r^2 = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}{(9.8\ \text{N/kg})}\sqrt{(45\text{ m})^2 - (0.5\text{ m})^2} \qquad r = \boxed{2.74\times10^{-4}\text{ m}}$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom. This equilibrium is unstable.

Section 13.2 **Free-Fall Acceleration and the Gravitational Force**

P13.9 $a = \frac{MG}{(4R_E)^2} = \frac{9.80\ \text{m/s}^2}{16.0} = \boxed{0.613\ \text{m/s}^2}$ toward the Earth.

***P13.10** (a) For the gravitational force on an object in the neighborhood of Miranda we have

$$m_{\text{obj}}g = \frac{Gm_{\text{obj}}m_{\text{Miranda}}}{r^2_{\text{Miranda}}}$$

$$g = \frac{Gm_{\text{Miranda}}}{r^2_{\text{Miranda}}} = \frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2(6.68\times10^{19}\text{ kg})}{\text{kg}^2(242\times10^3\text{ m})^2} = \boxed{0.0761\ \text{m/s}^2}$$

continued on next page

(b) We ignore the difference (of about 4%) in g between the lip and the base of the cliff. For the vertical motion of the athlete we have

$$y_f = y_i + v_{yi} + \frac{1}{2}a_y t^2$$

$$-5\ 000\ \text{m} = 0 + 0 + \frac{1}{2}\left(-0.076\,1\ \text{m/s}^2\right)t^2$$

$$t = \left(\frac{2(5\ 000\ \text{m})\text{s}^2}{0.076\,1\ \text{m}}\right)^{1/2} = \boxed{363\ \text{s}}$$

(c) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + (8.5\ \text{m/s})(363\ \text{s}) + 0 = \boxed{3.08 \times 10^3\ \text{m}}$

We ignore the curvature of the surface (of about 0.7°) over the athlete's trajectory.

(d) $v_{xf} = v_{xi} = 8.50\ \text{m/s}$

$v_{yf} = v_{yi} + a_y t = 0 - \left(0.076\,1\ \text{m/s}^2\right)(363\ \text{s}) = -27.6\ \text{m/s}$

Thus $\vec{\mathbf{v}}_f = \left(8.50\hat{\mathbf{i}} - 27.6\hat{\mathbf{j}}\right)\ \text{m/s} = \sqrt{8.5^2 + 27.6^2}\ \text{m/s at } \tan^{-1}\frac{27.6}{8.5}$ below the x axis.

$\boxed{\vec{\mathbf{v}}_f = 28.9\ \text{m/s at } 72.9° \text{ below the horizontal}}$

P13.11 $$g = \frac{GM}{R^2} = \frac{G\rho\left(4\pi R^3/3\right)}{R^2} = \frac{4}{3}\pi G\rho R$$

If

$$\frac{g_M}{g_E} = \frac{1}{6} = \frac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$$

then

$$\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}$$

Section 13.3 **Kepler's Laws and the Motion of Planets**

***P13.12** The particle does possess angular momentum, because it is not headed straight for the origin. Its angular momentum is constant because the object is free of outside influences.

Since speed is constant, the distance traveled between t_1 and t_2 is equal to the distance traveled between t_3 and t_4. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So

$$\frac{1}{2}bv\left(t_2 - t_1\right) = \frac{1}{2}bv\left(t_4 - t_3\right)$$

states that the particle's radius vector sweeps out equal areas in equal times.

P13.13 Applying Newton's 2nd Law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \qquad \text{or} \qquad M = \frac{4v^2 r}{G}$$

We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$. Therefore

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so,

$$M = \frac{2v^3 T}{\pi G} = \frac{2\left(220\times10^3 \text{ m/s}\right)^3 (14.4 \text{ d})(86\,400 \text{ s/d})}{\pi\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)} = \boxed{1.26\times10^{32} \text{ kg} = 63.3 \text{ solar masses}}$$

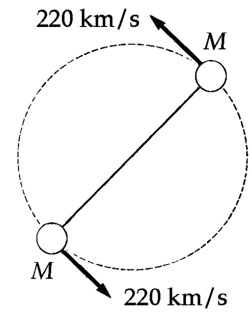

FIG. P13.13

P13.14 By Kepler's Third Law, $T^2 = ka^3$ (a = semi-major axis)
For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$. Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is
$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}}$ (out around the orbit of Pluto).

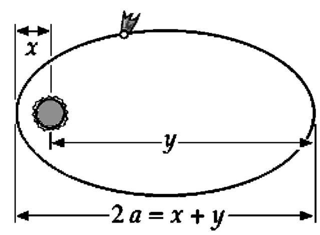

FIG. P13.14

P13.15 $T^2 = \dfrac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(4.22\times10^8 \text{ m}\right)^3}{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)(1.77\times86\,400 \text{ s})^2} = \boxed{1.90\times10^{27} \text{ kg}}$$

(approximately 316 Earth masses)

P13.16 $\sum F = ma$: $\dfrac{Gm_{\text{planet}} M_{\text{star}}}{r^2} = \dfrac{m_{\text{planet}} v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} \qquad \omega_y = \left(\frac{90.0°}{5.00 \text{ yr}}\right)3^{3/2} = \frac{468°}{5.00 \text{ yr}}$$

So $\boxed{\text{planet } Y \text{ has turned through 1.30 revolutions}}$.

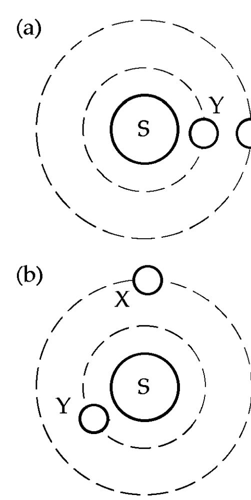

FIG. P13.16

P13.17 $$\frac{GM_J}{(R_J+d)^2}=\frac{4\pi^2(R_J+d)}{T^2}$$

$$GM_JT^2=4\pi^2(R_J+d)^3$$

$$(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27}\ \text{kg})(9.84\times3\,600)^2=4\pi^2(6.99\times10^7+d)^3$$

$d=\boxed{8.92\times10^7\ \text{m}}=\boxed{89\,200\ \text{km}}$ above the planet

P13.18 The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal acceleration:

$$\frac{GM_sm}{R_s^2}=\frac{mv^2}{R_s}=mR_s\omega^2$$

so

$$\omega=\sqrt{\frac{GM_s}{R_s^3}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left[2(1.99\times10^{30}\ \text{kg})\right]}{(10.0\times10^3\ \text{m})^3}}$$

$$\omega=\boxed{1.63\times10^4\ \text{rad/s}}$$

P13.19 The speed of a planet in a circular orbit is given by

$$\sum F=ma:\ \frac{GM_{\text{sun}}m}{r^2}=\frac{mv^2}{r}\qquad v=\sqrt{\frac{GM_{\text{sun}}}{r}}$$

For Mercury the speed is $$v_M=\sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.79\times10^{10})\ \text{s}^2}}=4.79\times10^4\ \text{m/s}$$

and for Pluto, $$v_P=\sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.91\times10^{12})\ \text{s}^2}}=4.74\times10^3\ \text{m/s}$$

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from the Sun after time t where

$$\sqrt{r_P^2+v_P^2t^2}=\sqrt{r_M^2+v_M^2t^2}$$

$$r_P^2-r_M^2=(v_M^2-v_P^2)t^2$$

$$t=\sqrt{\frac{(5.91\times10^{12}\ \text{m})^2-(5.79\times10^{10}\ \text{m})^2}{(4.79\times10^4\ \text{m/s})^2-(4.74\times10^3\ \text{m/s})^2}}=\sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}}=1.24\times10^8\ \text{s}=\boxed{3.93\ \text{yr}}$$

***P13.20** In $T^2=4\pi^2a^3/GM_{central}$ we take $a=3.84\times10^8$ m.

$$M_{central}=4\pi^2a^3/GT^2=\frac{4\pi^2(3.84\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86400\ \text{s})^2}=\boxed{6.02\times10^{24}\ \text{kg}}$$

This is a little larger than 5.98×10^{24} kg.

> The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth–Moon distance.

Section 13.4 **The Gravitational Field**

P13.21 $\vec{\mathbf{g}} = \frac{Gm}{l^2}\hat{\mathbf{i}} + \frac{Gm}{l^2}\hat{\mathbf{j}} + \frac{Gm}{2l^2}\left(\cos 45.0°\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$

so

$$\vec{\mathbf{g}} = \frac{GM}{l^2}\left(1+\frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$

or

$$\vec{\mathbf{g}} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2}+\frac{1}{2}\right) \text{ toward the opposite corner}}$$

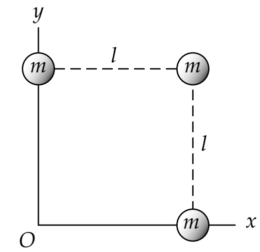

FIG. P13.21

P13.22 (a) $F = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\ \text{kg}\right)\left(10^3\ \text{kg}\right)\right]}{\left(1.00\times10^4\ \text{m}+50.0\ \text{m}\right)^2} = \boxed{1.31\times10^{17}\ \text{N}}$

(b) $\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

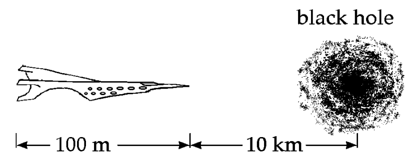

FIG. P13.22

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4\ \text{m}\right)^2 - \left(1.00\times10^4\ \text{m}\right)^2\right]}{\left(1.00\times10^4\ \text{m}\right)^2\left(1.01\times10^4\ \text{m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\ \text{N/kg}}$$

***P13.23** (a) $g_1 = g_2 = \frac{MG}{r^2+a^2}$

$g_{1y} = -g_{2y}$ $\qquad g_y = g_{1y} + g_{2y} = 0$

$g_{1x} = g_{2x} = g_2\cos\theta$ $\qquad \cos\theta = \frac{r}{\left(a^2+r^2\right)^{1/2}}$

$$\vec{\mathbf{g}} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or

$$\vec{\mathbf{g}} = \boxed{\frac{2MGr}{\left(r^2+a^2\right)^{3/2}} \text{ toward the center of mass}}$$

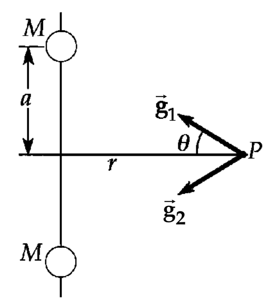

FIG. P13.23

(b) As r goes to zero, we approach the point halfway between the masses. Here the fields of the two are equally strong and in opposite directions so they add to zero.

(c) As $r \to 0$, $2MGr(r^2+a^2)^{-3/2}$ approaches $2MG(0)/a^3 = 0$

(d) Standing far away from the masses, their separateness makes no difference. They produce equal fields in the same direction to behave like a single object of mass $2M$.

(e) As r becomes much larger than a, the expression approaches $2MGr(r^2+0^2)^{-3/2} = 2MGr/r^3 = 2MG/r^2$ as required.

Section 13.5 **Gravitational Potential Energy**

P13.24 (a) $$U = -\frac{GM_E m}{r} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(100\ \text{kg})}{(6.37+2.00)\times10^6\ \text{m}} = \boxed{-4.77\times10^9\ \text{J}}$$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed $\boxed{\text{downward on the satellite and upward on the planet}}$.

$$F = \frac{GM_E m}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(100\ \text{kg})}{(8.37\times10^6\ \text{m})^2} = \boxed{569\ \text{N}}$$

P13.25 (a) $$\rho = \frac{M_S}{\frac{4}{3}\pi r_E^3} = \frac{3(1.99\times10^{30}\ \text{kg})}{4\pi(6.37\times10^6\ \text{m})^3} = \boxed{1.84\times10^9\ \text{kg/m}^3}$$

(b) $$g = \frac{GM_S}{r_E^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(6.37\times10^6\ \text{m})^2} = \boxed{3.27\times10^6\ \text{m/s}^2}$$

(c) $$U_g = -\frac{GM_S m}{r_E} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(1.00\ \text{kg})}{6.37\times10^6\ \text{m}} = \boxed{-2.08\times10^{13}\ \text{J}}$$

P13.26 The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$$K_i + U_i + \Delta E_{\text{mch}} = K_f + U_f: \qquad \frac{1}{2}M_p v_i^2 - \frac{GM_E M_p}{R_E} + 0 = 0 - \frac{GM_E M_p}{R_E + h}$$

$$\frac{1}{2}(10.0\times10^3\ \text{m/s})^2 - (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}}\right)$$

$$= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}+h}\right)$$

$$(5.00\times10^7\ \text{m}^2/\text{s}^2) - (6.26\times10^7\ \text{m}^2/\text{s}^2) = \frac{-3.99\times10^{14}\ \text{m}^3/\text{s}^2}{6.37\times10^6\ \text{m}+h}$$

$$6.37\times10^6\ \text{m}+h = \frac{3.99\times10^{14}\ \text{m}^3/\text{s}^2}{1.26\times10^7\ \text{m}^2/\text{s}^2} = 3.16\times10^7\ \text{m}$$

$$\boxed{h = 2.52\times10^7\ \text{m}}$$

***P13.27** (a) $$U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$$

$$U_{\text{Tot}} = -\frac{3(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.00\times10^{-3}\ \text{kg})^2}{0.300\ \text{m}} = \boxed{-1.67\times10^{-14}\ \text{J}}$$

(b) Each particle feels a net force of attraction toward the midpoint between the other two. Each moves toward the center of the triangle with the same acceleration. They collide simultaneously at the center of the triangle.

P13.28 $$W = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0\right)$$

$$W = \frac{(+6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\ \text{kg})(1.00\times10^3\ \text{kg})}{1.74\times10^6\ \text{m}} = \boxed{2.82\times10^9\ \text{J}}$$

P13.29 (a) Energy conservation of the object-Earth system from release to radius r:

$$\left(K+U_g\right)_{\text{altitude } h}=\left(K+U_g\right)_{\text{radius } r}$$

$$0-\frac{GM_Em}{R_E+h}=\frac{1}{2}mv^2-\frac{GM_Em}{r}$$

$$v=\left(2GM_E\left(\frac{1}{r}-\frac{1}{R_E+h}\right)\right)^{1/2}=-\frac{dr}{dt}$$

(b) $\int_i^f dt=\int_i^f -\frac{dr}{v}=\int_f^i \frac{dr}{v}$. The time of fall is

$$\Delta t=\int_{R_E}^{R_E+h}\left(2GM_E\left(\frac{1}{r}-\frac{1}{R_E+h}\right)\right)^{-1/2}dr$$

$$\Delta t=\int_{6.37\times10^6\text{ m}}^{6.87\times10^6\text{ m}}\left[2\times6.67\times10^{-11}\times5.98\times10^{24}\left(\frac{1}{r}-\frac{1}{6.87\times10^6\text{ m}}\right)\right]^{-1/2}dr$$

We can enter this expression directly into a mathematical calculation program. Alternatively, to save typing we can change variables to $u=\frac{r}{10^6}$. Then

$$\Delta t=\left(7.977\times10^{14}\right)^{-1/2}\int_{6.37}^{6.87}\left(\frac{1}{10^6u}-\frac{1}{6.87\times10^6}\right)^{-1/2}10^6\,du$$

$$=3.541\times10^{-8}\frac{10^6}{\left(10^6\right)^{-1/2}}\int_{6.37}^{6.87}\left(\frac{1}{u}-\frac{1}{6.87}\right)^{-1/2}du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall

$$\Delta t=3.541\times10^{-8}\times10^9\times9.596=339.8=\boxed{340\text{ s}}$$

Section 13.6 **Energy Considerations in Planetary and Satellite Motion**

P13.30 (a) $v_{\text{solar escape}}=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\cdot\text{Sun}}}}=\boxed{42.1\text{ km/s}}$

(b) Let $r=R_{E\cdot S}x$ represent variable distance from the Sun, with x in astronomical units.

$$v=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\cdot S}x}}=\frac{42.1}{\sqrt{x}}$$

If $v=\frac{125\,000\text{ km}}{3\,600\text{ s}}$, then $x=1.47\text{ A.U.}=\boxed{2.20\times10^{11}\text{ m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape).

P13.31 $\frac{1}{2}mv_i^2+GM_Em\left(\frac{1}{r_f}-\frac{1}{r_i}\right)=\frac{1}{2}mv_f^2$ $\qquad$ $\frac{1}{2}v_i^2+GM_E\left(0-\frac{1}{R_E}\right)=\frac{1}{2}v_f^2$

or

$$v_f^2=v_1^2-\frac{2GM_E}{R_E}$$

and

$$v_f=\left(v_1^2-\frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f=\left[\left(2.00\times10^4\right)^2-1.25\times10^8\right]^{1/2}=\boxed{1.66\times10^4\text{ m/s}}$$

***P13.32** $E_{\text{tot}} = -\dfrac{GMm}{2r}$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{2}\frac{10^3\ \text{kg}}{10^3\ \text{m}}\left(\frac{1}{6\,370+100} - \frac{1}{6\,370+200}\right)$$

$$\Delta E = 4.69\times10^8\ \text{J} = \boxed{469\ \text{MJ}}$$

> Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.

P13.33 To obtain the orbital velocity, we use $\sum F = \dfrac{mMG}{R^2} = \dfrac{mv^2}{R}$

or $v = \sqrt{\dfrac{MG}{R}}$

We can obtain the escape velocity from $\dfrac{1}{2}mv_{\text{esc}}^2 = \dfrac{mMG}{R}$

or $v_{\text{esc}} = \sqrt{\dfrac{2MG}{R}} = \boxed{\sqrt{2}v}$

***P13.34**

> Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket.

The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.495\times10^{23}\ \text{kg}\right)}{\text{kg}^2\left(2.64\times10^6\ \text{m}\right)}$$

$$U_1 = -3.78\times10^6 m_2\ \ \text{m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.90\times10^{27}\ \text{kg}\right)}{\text{kg}^2\left(1.071\times10^9\ \text{m}\right)}$$

$$U_2 = -1.18\times10^8 m_2\ \ \text{m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2}m_2v_{\text{esc}}^2 = +\left(3.78\times10^6 + 1.18\times10^8\right)m_2\ \ \text{m}^2/\text{s}^2$$

$$v_{\text{esc}} = \sqrt{2\times1.22\times10^8\ \text{m}^2/\text{s}^2} = \boxed{15.6\ \text{km/s}}$$

P13.35 $F_c = F_G$ gives $\dfrac{mv^2}{r} = \dfrac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\dfrac{GM_E}{r}}$

and period $= \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM_E}}$

(a) $r = R_E + 200\text{ km} = 6\,370\text{ km} + 200\text{ km} = 6\,570\text{ km}$

Thus,

$$\text{period} = 2\pi\left(6.57\times10^{6}\text{ m}\right)\sqrt{\frac{\left(6.57\times10^{6}\text{ m}\right)}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}}$$

$$T = 5.30\times10^{3}\text{ s} = 88.3\text{ min} = \boxed{1.47\text{ h}}$$

(b) $v = \sqrt{\dfrac{GM_E}{r}} = \sqrt{\dfrac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.57\times10^{6}\text{ m}\right)}} = \boxed{7.79\text{ km/s}}$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_E m}{r_f}\right) - \left(\frac{-GM_E m}{r_i}\right) \qquad (1)$$

$$r_i = R_E = 6.37\times10^{6}\text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400\text{ s}} = 4.63\times10^{2}\text{ m/s}$$

Substituting the appropriate values into (1) yields the

minimum energy input $= \boxed{6.43\times10^{9}\text{ J}}$

P13.36 The gravitational force supplies the needed centripetal acceleration.

Thus,

$$\frac{GM_E m}{\left(R_E + h\right)^2} = \frac{mv^2}{\left(R_E + h\right)} \qquad \text{or} \qquad v^2 = \frac{GM_E}{R_E + h}$$

(a) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi\left(R_E + h\right)}{\sqrt{\frac{GM_E}{(R_E+h)}}}$ $\qquad T = \boxed{2\pi\sqrt{\dfrac{\left(R_E + h\right)^3}{GM_E}}}$

(b) $v = \boxed{\sqrt{\dfrac{GM_E}{R_E + h}}}$

continued on next page

(c) Minimum energy input is $\Delta E_{\min} = \left(K_f + U_{gf}\right) - \left(K_i - U_{gi}\right)$

It is simplest to launch the satellite from a location on the equator, and launch it toward the east.

This choice has the object starting with energy $K_i = \frac{1}{2} m v_i^2$

with

$$v_i = \frac{2\pi R_E}{1.00 \text{ day}} = \frac{2\pi R_E}{86\,400 \text{ s}} \qquad \text{and} \qquad U_{gi} = -\frac{GM_E m}{R_E}$$

Thus,

$$\Delta E_{\min} = \frac{1}{2} m\left(\frac{GM_E}{R_E + h}\right) - \frac{GM_E m}{R_E + h} - \frac{1}{2} m\left[\frac{4\pi^2 R_E^2}{(86\,400 \text{ s})^2}\right] + \frac{GM_E m}{R_E}$$

or

$$\boxed{\Delta E_{\min} = GM_E m\left[\frac{R_E + 2h}{2R_E\left(R_E + h\right)}\right] - \frac{2\pi^2 R_E^2 m}{(86\,400 \text{ s})^2}}$$

P13.37 (a) Energy conservation for the object-Earth system from firing to apex:

$$\left(K + U_g\right)_i = \left(K + U_g\right)_f$$

$$\frac{1}{2} m v_i^2 - \frac{GmM_E}{R_E} = 0 - \frac{GmM_E}{R_E + h}$$

where $\frac{1}{2} m v_{\text{esc}}^2 = \frac{GmM_E}{R_E}$. Then

$$\frac{1}{2} v_i^2 - \frac{1}{2} v_{\text{esc}}^2 = -\frac{1}{2} v_{\text{esc}}^2 \frac{R_E}{R_E + h}$$

$$v_{\text{esc}}^2 - v_i^2 = \frac{v_{\text{esc}}^2 R_E}{R_E + h}$$

$$\frac{1}{v_{\text{esc}}^2 - v_i^2} = \frac{R_E + h}{v_{\text{esc}}^2 R_E}$$

$$h = \frac{v_{\text{esc}}^2 R_E}{v_{\text{esc}}^2 - v_i^2} - R_E = \frac{v_{\text{esc}}^2 R_E - v_{\text{esc}}^2 R_E + v_i^2 R_E}{v_{\text{esc}}^2 - v_i^2}$$

$$h = \frac{R_E v_i^2}{v_{\text{esc}}^2 - v_i^2}$$

(b) $h = \frac{6.37 \times 10^6 \text{ m } (8.76)^2}{(11.2)^2 - (8.76)^2} = \boxed{1.00 \times 10^7 \text{ m}}$

(c) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation

$$v_i^2 = v_{\text{esc}}^2\left(1 - \frac{R_E}{R_E + h}\right) = v_{\text{esc}}^2\left(\frac{h}{R_E + h}\right) = \left(11.2 \times 10^3 \text{ m/s}\right)^2\left(\frac{2.51 \times 10^7 \text{ m}}{6.37 \times 10^6 \text{ m} + 2.51 \times 10^7 \text{ m}}\right)$$

$$= 1.00 \times 10^8 \text{ m}^2/\text{s}^2$$

$$v_i = \boxed{1.00 \times 10^4 \text{ m/s}}$$

(d) With $v_i << v_{\text{esc}}$, $h \approx \frac{R_E v_i^2}{v_{\text{esc}}^2} = \frac{R_E v_i^2 R_E}{2GM_E}$. But $g = \frac{GM_E}{R_E^2}$, so $h = \frac{v_i^2}{2g}$, in agreement with $0^2 = v_i^2 + 2(-g)(h - 0)$

P13.38 (a) For the satellite $\sum F = ma$ $\qquad \dfrac{GmM_E}{r^2} = \dfrac{mv_0^2}{r}$

$$\boxed{v_0 = \left(\frac{GM_E}{r}\right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$\left(\sum mv\right)_i = \left(\sum mv\right)_f$$

$$5mv_0 = 4mv_i + m0$$

$$v_i = \frac{5}{4}v_0 = \boxed{\frac{5}{4}\left(\frac{GM_E}{r}\right)^{1/2}}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to r and v_i by $4mrv_i = 4mr_f v_f$ and

$\dfrac{1}{2}4mv_i^2 - \dfrac{GM_E 4m}{r} = \dfrac{1}{2}4mv_f^2 - \dfrac{GM_E 4m}{r_f}$. Substituting $v_f = \dfrac{v_i r}{r_f}$ we have

$\dfrac{1}{2}v_i^2 - \dfrac{GM_E}{r} = \dfrac{1}{2}\dfrac{v_i^2 r^2}{r_f^2} - \dfrac{GM_E}{r_f}$. Further, substituting $v_i^2 = \dfrac{25}{16}\dfrac{GM_E}{r}$ gives

$$\frac{25}{32}\frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32}\frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$

$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing of fractions, $-7r_f^2 = 25r^2 - 32rr_f$ or $7\left(\dfrac{r_f}{r}\right)^2 - 32\left(\dfrac{r_f}{r}\right) + 25 = 0$ giving

$\dfrac{r_f}{r} = \dfrac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \dfrac{50}{14}$ or $\dfrac{14}{14}$. The latter root describes the starting point. The outer end of the orbit has $\dfrac{r_f}{r} = \dfrac{25}{7}$; $\boxed{r_f = \dfrac{25r}{7}}$

P13.39 (a) The major axis of the orbit is $2a = 50.5$ AU so $a = 25.25$ AU
Further, in the textbook's diagram of an ellipse, $a + c = 50$ AU so $c = 24.75$ AU

Then

$$e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\text{ yr})^2 = K_s(1\text{ AU})^3 \qquad K_s = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}$$

Then

$$T^2 = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}(25.25\text{ AU})^3 \qquad T = \boxed{127\text{ yr}}$$

(c) $$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(1.2\times10^{10}\text{ kg}\right)}{50\left(1.496\times10^{11}\text{ m}\right)} = \boxed{-2.13\times10^{17}\text{ J}}$$

Additional Problems

***P13.40** (a) Let R represent the radius of the asteroid. Then its volume is $\frac{4}{3}\pi R^3$ and its mass is $\rho\frac{4}{3}\pi R^3$.

For your orbital motion, $\sum F = ma$, $\quad \dfrac{Gm_1m_2}{R^2} = \dfrac{m_2v^2}{R}$, $\quad \dfrac{G\rho 4\pi R^3}{3R^2} = \dfrac{v^2}{R}$

$$R = \left(\frac{3v^2}{G\rho 4\pi}\right)^{1/2} = \left(\frac{3(8.5\ \text{m/s})^2\ \text{kg}^2\ \text{m}^3}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2(1\,100\ \text{kg})4\pi}\right)^{1/2} = \boxed{1.53\times10^4\ \text{m}}$$

(b) $\rho\frac{4}{3}\pi R^3 = (1\,100\ \text{kg/m}^3)\frac{4}{3}\pi(1.53\times10^4\ \text{m})^3 = \boxed{1.66\times10^{16}\ \text{kg}}$

(c) $v = \dfrac{2\pi R}{T} \qquad T = \dfrac{2\pi R}{v} = \dfrac{2\pi(1.53\times10^4\ \text{m})}{8.5\ \text{m/s}} = \boxed{1.13\times10^4\ \text{s}} = 3.15\ \text{h}$

(d) For an illustrative model, we take your mass as 90 kg and assume the asteroid is originally at rest. Angular momentum is conserved for the asteroid-you system:

$$\sum L_i = \sum L_f$$

$$0 = m_2vR - I\omega$$

$$0 = m_2vR - \frac{2}{5}m_1R^2\frac{2\pi}{T_{\text{asteroid}}}$$

$$m_2v = \frac{4\pi}{5}\frac{m_1R}{T_{\text{asteroid}}}$$

$$T_{\text{asteroid}} = \frac{4\pi m_1R}{5m_2v} = \frac{4\pi(1.66\times10^{16}\ \text{kg})(1.53\times10^4\ \text{m})}{5(90\ \text{kg})(8.5\ \text{m/s})} = 8.37\times10^{17}\ \text{s} = 26.5\ \text{billion years}$$

Thus your running does not produce significant rotation of the asteroid if it is originally stationary, and does not significantly affect any rotation it does have.

This problem is realistic. Many asteroids, such as Ida and Eros, are roughly 30 km in diameter. They are typically irregular in shape and not spherical. Satellites such as Phobos (of Mars), Adrastea (of Jupiter), Calypso (of Saturn), and Ophelia (of Uranus) would allow a visitor the same experience of easy orbital motion. So would many Kuiper-belt objects.

P13.41 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $F_s = \dfrac{GM_s m}{(r_E - x)^2}$

while the Earth exerts on it a radial outward force of $F_E = \dfrac{GM_E m}{x^2}$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus,

$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to

$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or

$$5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or

$$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

P13.42 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

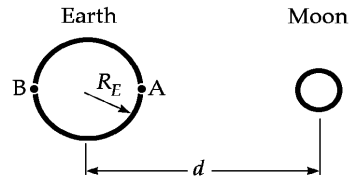

FIG. P13.42

$$a = G\frac{M_{\text{Moon}}}{d^2}$$

At the point A nearest the Moon, $a_+ = G\dfrac{M_M}{(d-r)^2}$

At the point B farthest from the Moon, $a_- = G\dfrac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \dfrac{2GM_M r}{d^3} = 1.11\times10^{-6}\ \text{m/s}^2$

Across the planet, $\dfrac{\Delta g}{g} = \dfrac{2\Delta a}{g} = \dfrac{2.22\times10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26\times10^{-7}}$

P13.43 Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \qquad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected impulse is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

P13.44 Momentum is conserved:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = M\vec{\mathbf{v}}_{1f} + 2M\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = -\frac{1}{2}\vec{\mathbf{v}}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

P13.45 (a) Each bit of mass dm in the ring is at the same distance from the object at A. The separate contributions $-\frac{Gmdm}{r}$ to the system energy add up to $-\frac{GmM_{\text{ring}}}{r}$. When the object is at A, this is

$$\frac{-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\ 1\,000\ \text{kg}\ 2.36\times10^{20}\ \text{kg}}{\text{kg}^2\sqrt{(1\times10^8\ \text{m})^2+(2\times10^8\ \text{m})^2}}=\boxed{-7.04\times10^4\ \text{J}}$$

(b) When the object is at the center of the ring, the potential energy is

$$-\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\ 1\,000\ \text{kg}\ 2.36\times10^{20}\ \text{kg}}{\text{kg}^2 1\times10^8\ \text{m}}=\boxed{-1.57\times10^5\ \text{J}}$$

(c) Total energy of the object-ring system is conserved:

$$(K+U_g)_A=(K+U_g)_B$$

$$0-7.04\times10^4\ \text{J}=\frac{1}{2}1\,000\ \text{kg}v_B^2-1.57\times10^5\ \text{J}$$

$$v_B=\left(\frac{2\times8.70\times10^4\ \text{J}}{1\,000\ \text{kg}}\right)^{1/2}=\boxed{13.2\ \text{m/s}}$$

P13.46 (a) The free-fall acceleration produced by the Earth is $g=\frac{GM_E}{r^2}=GM_Er^{-2}$ (directed downward)

Its rate of change is

$$\frac{dg}{dr}=GM_E(-2)r^{-3}=-2GM_Er^{-3}$$

The minus sign indicates that g decreases with increasing height.

At the Earth's surface,

$$\boxed{\frac{dg}{dr}=-\frac{2GM_E}{R_E^3}}$$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r}=\frac{|\Delta g|}{h}=\frac{2GM_E}{R_E^3}$$

Thus,

$$\boxed{|\Delta g|=\frac{2GM_Eh}{R_E^3}}$$

(c) $$|\Delta g|=\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(6.00\ \text{m})}{(6.37\times10^6\ \text{m})^3}=\boxed{1.85\times10^{-5}\ \text{m/s}^2}$$

P13.47 From the walk, $2\pi r=25\,000$ m. Thus, the radius of the planet is $r=\frac{25\,000\ \text{m}}{2\pi}=3.98\times10^3$ m

From the drop: $\Delta y=\frac{1}{2}gt^2=\frac{1}{2}g(29.2\ \text{s})^2=1.40$ m

so,

$$g=\frac{2(1.40\ \text{m})}{(29.2\ \text{s})^2}=3.28\times10^{-3}\ \text{m/s}^2=\frac{MG}{r^2}\qquad\therefore M=\boxed{7.79\times10^{14}\ \text{kg}}$$

P13.48 The distance between the orbiting stars is $d = 2r\cos 30° = \sqrt{3}r$ since $\cos 30° = \frac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

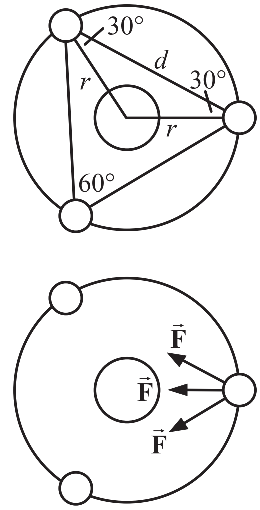

FIG. P13.48

$$\frac{Gmm}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gmm}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\frac{Gm2\cos 30°}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2 r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}} + M\right) = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{G\left(M + m/\sqrt{3}\right)}$$

$$T = 2\pi\left(\frac{r^3}{G\left(M + m/\sqrt{3}\right)}\right)^{1/2}$$

P13.49 For a 6.00 km diameter cylinder, $r = 3\,000$ m and to simulate $1g = 9.80\ \text{m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.057\,2\ \text{rad/s}}$$

The required rotation rate of the cylinder is $\boxed{\frac{1\ \text{rev}}{110\ \text{s}}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

P13.50 For both circular orbits,

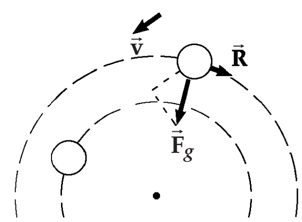

FIG. P13.50

$$\sum F = ma: \qquad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

(a) The original speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m} + 2\times10^5\ \text{m}\right)}} = \boxed{7.79\times10^3\ \text{m/s}}$

(b) The final speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.47\times10^6\ \text{m}\right)}} = \boxed{7.85\times10^3\ \text{m/s}}$

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_E m}{2r}$$

(c) Originally $E_i = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{2\left(6.57\times10^6\ \text{m}\right)} = \boxed{-3.04\times10^9\ \text{J}}$

continued on next page

(d) Finally $$E_f = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(100\ \text{kg})}{2(6.47\times10^{6}\ \text{m})} = \boxed{-3.08\times10^{9}\ \text{J}}$$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04\times10^{9}\ \text{J} - (-3.08\times10^{9}\ \text{J}) = \boxed{4.69\times10^{7}\ \text{J}}$$

(f) The only forces on the object are the backward force of air resistance R, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, one component of the gravitational force pulls forward on the satellite to do positive work and make its speed increase.

P13.51 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \quad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore,

$$0 = m_1v_1 - m_2v_2 \quad (2)$$

Combine equations (1) and (2):

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}} \quad \text{and} \quad \boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$$

Relative velocity

$$v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$$v_1 = 1.03\times10^{4}\ \text{m/s} \quad \text{and} \quad v_2 = 2.58\times10^{3}\ \text{m/s}$$

Therefore,

$$K_1 = \frac{1}{2}m_1v_1^2 = \boxed{1.07\times10^{32}\ \text{J}} \quad \text{and} \quad K_2 = \frac{1}{2}m_2v_2^2 = \boxed{2.67\times10^{31}\ \text{J}}$$

P13.52 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_av_a = mr_pv_p \text{ and } v_a = v_p\left(\frac{r_p}{r_a}\right) = (3.027\times10^{4}\ \text{m/s})\left(\frac{1.471}{1.521}\right) = \boxed{2.93\times10^{4}\ \text{m/s}}$$

(b) $$K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}(5.98\times10^{24})(3.027\times10^{4})^2 = \boxed{2.74\times10^{33}\ \text{J}}$$

$$U_p = -\frac{GmM}{r_p} = -\frac{(6.673\times10^{-11})(5.98\times10^{24})(1.99\times10^{30})}{1.471\times10^{11}} = \boxed{-5.40\times10^{33}\ \text{J}}$$

(c) Using the same form as in part (b), $K_a = \boxed{2.57\times10^{33}\ \text{J}}$ and $U_a = \boxed{-5.22\times10^{33}\ \text{J}}$.

Compare to find that $K_p + U_p = \boxed{-2.66\times10^{33}\ \text{J}}$ and $K_a + U_a = \boxed{-2.65\times10^{33}\ \text{J}}$. They agree.

P13.53 (a) $T=\dfrac{2\pi r}{v}=\dfrac{2\pi\left(30\,000\times 9.46\times 10^{15}\ \text{m}\right)}{2.50\times 10^{5}\ \text{m/s}}=7\times 10^{15}\ \text{s}=\boxed{2\times 10^{8}\ \text{yr}}$

(b) $M=\dfrac{4\pi^2 a^3}{GT^2}=\dfrac{4\pi^2\left(30\,000\times 9.46\times 10^{15}\ \text{m}\right)^3}{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.13\times 10^{15}\ \text{s}\right)^2}=2.66\times 10^{41}\ \text{kg}$

$M=1.34\times 10^{11}$ solar masses $\boxed{\sim 10^{11}\ \text{solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

P13.54 Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r}=\frac{M_c G}{r^2}=\frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2=4\pi^2 r^3$$

$$\left(6.67\times 10^{-11}\right)(20)\left(1.99\times 10^{30}\right)\left(5.00\times 10^{-3}\right)^2=4\pi^2 r^3$$

$$r_{\text{orbit}}=\boxed{119\ \text{km}}$$

P13.55 Let m represent the mass of the meteoroid and v_i its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

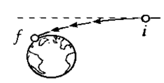

FIG. P13.55

$L_i=L_f$: $\quad m\vec{\mathbf{r}}_i\times\vec{\mathbf{v}}_i=m\vec{\mathbf{r}}_f\times\vec{\mathbf{v}}_f$

$$m\left(3R_E v_i\right)=mR_E v_f$$

$$v_f=3v_i$$

Now energy of the meteoroid-Earth system is also conserved:

$\left(K+U_g\right)_i=\left(K+U_g\right)_f$: $\quad \dfrac{1}{2}mv_i^2+0=\dfrac{1}{2}mv_f^2-\dfrac{GM_E m}{R_E}$

$$\frac{1}{2}v_i^2=\frac{1}{2}\left(9v_i^2\right)-\frac{GM_E}{R_E}$$

$\dfrac{GM_E}{R_E}=4v_i^2$: $\quad \boxed{v_i=\sqrt{\dfrac{GM_E}{4R_E}}}$

P13.56 (a) From the data about perigee, the energy of the satellite-Earth system is

$$E=\frac{1}{2}mv_p^2-\frac{GM_E m}{r_p}=\frac{1}{2}(1.60)\left(8.23\times 10^{3}\right)^2-\frac{\left(6.67\times 10^{-11}\right)\left(5.98\times 10^{24}\right)(1.60)}{7.02\times 10^{6}}$$

or

$$E=\boxed{-3.67\times 10^{7}\ \text{J}}$$

(b) $L=mvr\sin\theta=mv_p r_p\sin 90.0°=(1.60\ \text{kg})\left(8.23\times 10^{3}\ \text{m/s}\right)\left(7.02\times 10^{6}\ \text{m}\right)$

$=\boxed{9.24\times 10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}$

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(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $mv_a r_a \sin 90.0° = L$

Thus, $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)}{r_a} = -3.67\times10^7 \text{ J}$

and $(1.60 \text{ kg})v_a r_a = 9.24\times10^{10} \text{ kg}\cdot\text{m}^2/\text{s}$

Solving simultaneously, $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)(1.60)v_a}{9.24\times10^{10}}$

$= -3.67\times10^7$

which reduces to $0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$

so $v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)(3.672\,3\times10^7)}}{2(0.800)}$

This gives $v_a = 8\,230 \text{ m/s}$ or $\boxed{5\,580 \text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus,

$$r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10} \text{ kg}\cdot\text{m}^2/\text{s}}{(1.60 \text{ kg})(5.58\times10^3 \text{ m/s})} = \boxed{1.04\times10^7 \text{ m}}$$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}(7.02\times10^6 \text{ m} + 1.04\times10^7 \text{ m}) = \boxed{8.69\times10^6 \text{ m}}$$

(e) $T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2(8.69\times10^6 \text{ m})^3}{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24} \text{ kg})}}$

$T = 8\,060 \text{ s} = \boxed{134 \text{ min}}$

P13.57 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \quad \text{and} \quad Mr_2 = mr_1$$

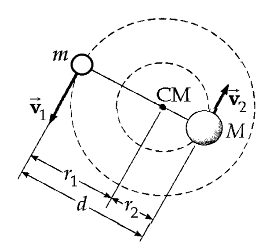

FIG. P13.57

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \quad \text{and} \quad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives $(r_1 + r_2)\omega^2 = \frac{(M+m)G}{d^2}$ with

$$\omega_1 = \omega_2 = \omega$$

and

$$T = \frac{2\pi}{\omega}$$

we find

$$\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$$

P13.58 From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Problem 33 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius R of the planet.

$$\sum F = ma: \qquad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$G\rho V = \frac{R^2\left(4\pi^2 R^2\right)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi$ $\qquad \boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

***P13.59** The gravitational forces the particles exert on each other are in the x direction. They do not affect the velocity of the center of mass. Energy is conserved for the pair of particles in a reference frame coasting along with their center of mass, and momentum conservation means that the identical particles move toward each other with equal speeds in this frame:

$$U_{gi} + K_i + K_i = U_{gf} + K_f + K_f$$

$$-\frac{Gm_1 m_2}{r_i} + 0 = -\frac{Gm_1 m_2}{r_f} + \tfrac{1}{2}m_1 v^2 + \tfrac{1}{2}m_2 v^2$$

$$-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1000\ \text{kg})^2}{20\ \text{m}} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1000\ \text{kg})^2}{2\ \text{m}} + 2\left(\tfrac{1}{2}\right)(1000\ \text{kg})v^2$$

$$\left(\frac{3.00\times10^{-5}\ \text{J}}{1000\ \text{kg}}\right)^{1/2} = v = 1.73\times10^{-4}\ \text{m/s}$$

Then their vector velocities are $\boxed{(800 + 1.73\times10^{-4})\,\hat{\mathbf{i}}\ \text{m/s and } (800 - 1.73\times10^{-4})\,\hat{\mathbf{i}}\ \text{m/s}}$ for the trailing particle and the leading particle, respectively.

***P13.60** (a) The gravitational force exerted on m by the Earth (mass ME) accelerates m according to $mg_2 = \frac{GmM_E}{r^2}$. The equal magnitude force exerted on the Earth by m produces acceleration of the Earth given by $g_1 = \frac{Gm}{r^2}$. The acceleration of relative approach is then

$$g_2 + g_1 = \frac{Gm}{r^2} + \frac{GM_E}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg} + m\right)}{\left(1.20\times10^7\ \text{m}\right)^2}$$

$$= \boxed{\left(2.77\ \text{m/s}^2\right)\left(1 + \frac{m}{5.98\times10^{24}\ \text{kg}}\right)}$$

(b) and (c) Here $m = 5$ kg and $m = 2000$ kg are both negligible compared to the mass of the Earth, so the acceleration of relative approach is just $\boxed{2.77\ \text{m/s}^2}$.

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(d) Again, m accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward m with an acceleration given as

$$M_E g_1 = \frac{GM_E m}{r^2}$$

$$g_1 = \frac{Gm}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.00\times10^{24}\ \text{kg}\right)}{\left(1.20\times10^{7}\ \text{m}\right)^2} = 0.926\ \text{m/s}^2$$

The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}$$

(e) Any object with mass small compared to the Earth starts to fall with acceleration $2.77\ \text{m/s}^2$. As m increases to become comparable to the mass of the Earth, the acceleration increases, and can become arbitrarily large. It approaches a direct proportionality to m.

P13.61 For the Earth, $\sum F = ma$: $\quad \dfrac{GM_s m}{r^2} = \dfrac{mv^2}{r} = \dfrac{m}{r}\left(\dfrac{2\pi r}{T}\right)^2$

Then $\quad GM_s T^2 = 4\pi^2 r^3$

Also the angular momentum $\quad L = mvr = m\dfrac{2\pi r}{T} r$ is a constant for the Earth.

We eliminate $\quad r = \sqrt{\dfrac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \qquad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}$$

Now the rates of change with time t are described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0 \qquad \frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{\Delta t}$$

$$\Delta T \approx -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) = -5\,000\ \text{yr}\left(\frac{3.16\times10^7\ \text{s}}{1\ \text{yr}}\right)\left(-3.64\times10^9\ \text{kg/s}\right)\left(2\frac{1\ \text{yr}}{1.991\times10^{30}\ \text{kg}}\right)$$

$$\Delta T = \boxed{1.82\times10^{-2}\ \text{s}}$$

ANSWERS TO EVEN PROBLEMS

P13.2 $2.67\times10^{-7}\ \text{m/s}^2$

P13.4 3.00 kg and 2.00 kg

P13.6 (a) 4.39×10^{20} N toward the Sun (b) 1.99×10^{20} N toward the Earth (c) 3.55×10^{22} N toward the Sun (d) Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.8 There are two possibilities: either $1\ \text{m} - 61.3\ \text{nm}$ or $2.74\times10^{-4}\ \text{m}$

P13.10 (a) 7.61 cm/s^2 (b) 363 s (c) 3.08 km (d) 28.9 m/s at 72.9° below the horizontal

P13.12 The particle does possess angular momentum, because it is not headed straight for the origin. Its angular momentum is constant because the object is free of outside influences. See the solution.

P13.14 35.2 AU

P13.16 Planet *Y* has turned through 1.30 revolutions.

P13.18 1.63×10^{4} rad/s

P13.20 6.02×10^{24} kg. The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth–Moon distance.

P13.22 (a) 1.31×10^{17} N toward the center (b) 2.62×10^{12} N/kg

P13.24 (a) -4.77×10^{9} J (b) 569 N down (c) 569 N up

P13.26 2.52×10^{7} m

P13.28 2.82×10^{9} J

P13.30 (a) 42.1 km/s (b) 2.20×10^{11} m

P13.32 469 MJ. Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.

P13.34 Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket. 15.6 km/s

P13.36 (a) $2\pi\left(R_E+h\right)^{3/2}\left(GM_E\right)^{-1/2}$ (b) $\left(GM_E\right)^{1/2}\left(R_E+h\right)^{-1/2}$ (c) $GM_E m\left[\dfrac{R_E+2h}{2R_E\left(R_E+h\right)}\right]-\dfrac{2\pi^2R_E^2m}{(86\,400\text{ s})^2}$

The satellite should be launched from the Earth's equator toward the east.

P13.38 (a) $v_0=\left(\dfrac{GM_E}{r}\right)^{1/2}$ (b) $v_i=\dfrac{5\left(GM_E/r\right)^{1/2}}{4}$ (c) $r_f=\dfrac{25r}{7}$

P13.40 (a) 15.3 km (b) 1.66×10^{16} kg (c) 1.13×10^{4} s (d) No. Its mass is so large compared with mine that I would have negligible effect on its rotation.

P13.42 2.26×10^{-7}

P13.44 $\dfrac{2}{3}\sqrt{\dfrac{GM}{R}}$; $\dfrac{1}{3}\sqrt{\dfrac{GM}{R}}$

P13.46 (a), (b) see the solution (c) 1.85×10^{-5} m/s^2

P13.48 see the solution

P13.50 (a) 7.79 km/s (b) 7.85 km/s (c) −3.04 GJ (d) −3.08 GJ (e) loss = 46.9 MJ
(f) A component of the Earth's gravity pulls forward on the satellite in its downward banking trajectory.

P13.52 (a) 29.3 km/s (b) $K_p = 2.74 \times 10^{33}$ J; $U_p = -5.40 \times 10^{33}$ J
(c) $K_a = 2.57 \times 10^{33}$ J; $U_a = -5.22 \times 10^{33}$ J; yes

P13.54 119 km

P13.56 (a) −36.7 MJ (b) 9.24×10^{10} kg·m^2/s (c) 5.58 km/s; 10.4 Mm (d) 8.69 Mm (e) 134 min

P13.58 see the solution

P13.60 (a) (2.77 m/s^2)(1 + m/5.98 × 10^{24} kg) (b) 2.77 m/s^2 (c) 2.77 m/s^2 (d) 3.70 m/s^2
(e) Any object with mass small compared with the mass of the Earth starts to fall with acceleration 2.77 m/s^2. As m increases to become comparable to the mass of the Earth, the acceleration increases and can become arbitrarily large. It approaches a direct proportionality to m.