

22

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

- 22.1 Heat Engines and the Second Law of Thermodynamics
- 22.2 Heat Pumps and Refrigerators
- 22.3 Reversible and Irreversible Processes
- 22.4 The Carnot Engine
- 22.5 Gasoline and Diesel Engines
- 22.6 Entropy
- 22.7 Entropy Changes in Irreversible Processes
- 22.8 Entropy on a Microscopic Scale

ANSWERS TO QUESTIONS

- Q22.1** First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.
- *Q22.2** For any cyclic process the total input energy must be equal to the total output energy. This is a consequence of the first law of thermodynamics. It is satisfied by processes ii, iv, v, vi, vii but not by processes i, iii, viii. The second law says that a cyclic process that takes in energy by heat must put out some of the energy by heat. This is not satisfied for processes v, vii, and viii. Thus the answers are (i) b (ii) a (iii) b (iv) a (v) c (vi) a (vii) c (viii) d.
- Q22.3** A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c , and steam at T_h , the efficiency of the power plant goes as $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high T_h .
- Q22.4** No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.
- Q22.5** Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. The chemical energy turning into internal energy can be modeled as energy input by heat. The second law says that not all of the energy input can become output mechanical energy. Much of the input energy must and does become energy output by heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat in the atmosphere.

- *Q22.6** Answer (b). In the reversible adiabatic expansion OA , the gas does work against a piston, takes in no energy by heat, and so drops in internal energy and in temperature. In the free adiabatic expansion OB , there is no piston, no work output, constant internal energy, and constant temperature for the ideal gas. The points O and B are on a hyperbolic isotherm. The points O and A are on an adiabat, steeper than an isotherm by the factor γ .
- Q22.7** A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.
- Q22.8** (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.
- (b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.
- *Q22.9** (i) Answer (a). The air conditioner operating in a closed room takes in energy by electric transmission and turns it all into energy put out by heat. That is its whole net effect.
- (ii) Answer (b). The frozen stuff absorbs energy by heat from the air. But if you fill the ice trays with tap water and put them back into the freezer, the refrigerator will pump more heat into the air than it extracts from the water to make it freeze.
- *Q22.10** (i) Answer (d). (ii) Answer (d). The second law says that you must put in some work to pump heat from a lower-temperature to a higher-temperature location. But it can be very little work if the two temperatures are very nearly equal.
- Q22.11** One: Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff.
- Two: As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change.
- Three: The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.
- Q22.12** (a) For an expanding ideal gas at constant temperature, the internal energy stays constant. The gas must absorb by heat the same amount of energy that it puts out by work. Then its entropy change is
$$\Delta S = \frac{\Delta Q}{T} = nR \ln \left(\frac{V_2}{V_1} \right)$$
- (b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).
- *Q22.13** Answer (f). The whole Universe must have an entropy change of zero or more. The environment around the system comprises the rest of the Universe, and must have an entropy change of $+8.0 \text{ J/K}$, or more.

- *Q22.14** (i) Consider the area that fits under each of the arrows, between its line segment and the horizontal axis. Count it as positive for arrows to the right, zero for vertical arrows, and negative for arrows tending left. Then $E > F > G > H = D > A > B > C$.
- (ii) The thin blue hyperbolic lines are isotherms. Each is a set of points representing states with the same internal energy for the ideal gas simple. An arrow tending farther from the origin than the BE hyperbola represents a process for which internal energy increases. So we have $D = E > C > B = F > G > A = H$.
- (iii) The arrows C and G are along an adiabat. Visualize or sketch in a set of these curves, uniformly steeper than the blue isotherms. The energy input by heat is determined by how far above the starting adiabat the process arrow ends. We have $E > D > F > C = G > B > H > A$.
- *Q22.15** Processes C and G are adiabatic. They can be carried out reversibly. Along these arrows entropy does not change. Visualize or sketch in a set of these adiabatic curves, uniformly steeper than the blue isotherms. The entropy change is determined by how far above the starting adiabat the process arrow ends. We have $E > D > F > C = G > B > H > A$.
- *Q22.16** (a) The reduced flow rate of 'cooling water' reduces the amount of heat exhaust Q_c that the plant can put out each second. Even with constant efficiency, the rate at which the turbines can take in heat is reduced and so is the rate at which they can put out work to the generators. If anything, the efficiency will drop, because the smaller amount of water carrying the heat exhaust will tend to run hotter. The steam going through the turbines will undergo a smaller temperature change. Thus there are two reasons for the work output to drop.
- (b) The engineer's version of events, as seen from inside the plant, is complete and correct. Hot steam pushes hard on the front of a turbine blade. Still-warm steam pushes less hard on the back of the blade, which turns in response to the pressure difference. Higher temperature at the heat exhaust port in the lake works its way back to a corresponding higher temperature of the steam leaving a turbine blade, a smaller temperature drop across the blade, and a lower work output.
- *Q22.17** Answer (d). Heat input will not *necessarily* produce an entropy increase, because a heat input could go on simultaneously with a larger work output, to carry the gas to a lower-temperature, lower-entropy final state. Work input will not *necessarily* produce an entropy increase, because work input could go on simultaneously with heat output to carry the gas to a lower-volume, lower-entropy final state. Either temperature increase at constant volume, or volume increase at constant temperature, or simultaneous increases in both temperature and volume, will necessarily end in a more disordered, higher-entropy final state.
- Q22.18** An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.

Q22.19 Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy which it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed. Arnold Sommerfeld suggested the idea for this question.

Q22.20 Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

SOLUTIONS TO PROBLEMS

Section 22.1 Heat Engines and the Second Law of Thermodynamics

P22.1 (a) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{25.0 \text{ J}}{360 \text{ J}} = \boxed{0.0694}$ or $\boxed{6.94\%}$

(b) $|Q_c| = |Q_h| - W_{\text{eng}} = 360 \text{ J} - 25.0 \text{ J} = \boxed{335 \text{ J}}$

***P22.2** The engine's output work we identify with the kinetic energy of the bullet:

$$W_{\text{eng}} = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.0024 \text{ kg})(320 \text{ m/s})^2 = 123 \text{ J}$$

$$e = \frac{W_{\text{eng}}}{Q_h}$$

$$Q_h = \frac{W_{\text{eng}}}{e} = \frac{123 \text{ J}}{0.011} = 1.12 \times 10^4 \text{ J}$$

$$Q_h = W_{\text{eng}} + |Q_c|$$

The energy exhaust is

$$|Q_c| = Q_h - W_{\text{eng}} = 1.12 \times 10^4 \text{ J} - 123 \text{ J} = 1.10 \times 10^4 \text{ J}$$

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{1.10 \times 10^4 \text{ J kg}^\circ\text{C}}{1.80 \text{ kg } 448 \text{ J}} = \boxed{13.7^\circ\text{C}}$$

P22.3 (a) We have $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 0.250$

with $|Q_c| = 8000 \text{ J}$, we have $|Q_h| = \boxed{10.7 \text{ kJ}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = 2667 \text{ J}$

and from $\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t = \frac{W_{\text{eng}}}{\mathcal{P}} = \frac{2667 \text{ J}}{5000 \text{ J/s}} = \boxed{0.533 \text{ s}}$

P22.4 (a) The input energy each hour is

$$(7.89 \times 10^3 \text{ J/revolution})(2500 \text{ rev/min}) \frac{60 \text{ min}}{1 \text{ h}} = 1.18 \times 10^9 \text{ J/h}$$

$$\text{implying fuel input } (1.18 \times 10^9 \text{ J/h}) \left(\frac{1 \text{ L}}{4.03 \times 10^7 \text{ J}} \right) = \boxed{29.4 \text{ L/h}}$$

(b) $Q_h = W_{\text{eng}} + |Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t} + \frac{|Q_c|}{\Delta t}$$

$$\text{Useful power output} = \frac{W_{\text{eng}}}{\Delta t} = \frac{Q_h}{\Delta t} - \frac{|Q_c|}{\Delta t}$$

$$= \left(\frac{7.89 \times 10^3 \text{ J}}{\text{revolution}} - \frac{4.58 \times 10^3 \text{ J}}{\text{revolution}} \right) \frac{2500 \text{ rev}}{1 \text{ min}} \frac{1 \text{ min}}{60 \text{ s}} = 1.38 \times 10^5 \text{ W}$$

$$P_{\text{eng}} = 1.38 \times 10^5 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = \boxed{185 \text{ hp}}$$

$$(c) P_{\text{eng}} = \tau \omega \Rightarrow \tau = \frac{P_{\text{eng}}}{\omega} = \frac{1.38 \times 10^5 \text{ J/s}}{(2500 \text{ rev}/60 \text{ s})} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = \boxed{527 \text{ N} \cdot \text{m}}$$

$$(d) \frac{|Q_c|}{\Delta t} = \frac{4.58 \times 10^3 \text{ J}}{\text{revolution}} \left(\frac{2500 \text{ rev}}{60 \text{ s}} \right) = \boxed{1.91 \times 10^5 \text{ W}}$$

P22.5 The heat to melt 15.0 g of Hg is $|Q_c| = mL_f = (15 \times 10^{-3} \text{ kg})(1.18 \times 10^4 \text{ J/kg}) = 177 \text{ J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h| = mL_f = (10^{-3} \text{ kg})(3.97 \times 10^5 \text{ J/kg}) = 397 \text{ J}$$

and the work output is

$$W_{\text{eng}} = |Q_h| - |Q_c| = 220 \text{ J}$$

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{220 \text{ J}}{397 \text{ J}} = 0.554, \text{ or } \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h - T_c}{T_h} = \frac{933 \text{ K} - 243.1 \text{ K}}{933 \text{ K}} = 0.749 = 74.9\%$

Section 22.2 Heat Pumps and Refrigerators

P22.6 $\text{COP}(\text{refrigerator}) = \frac{Q_c}{W}$

(a) If $Q_c = 120 \text{ J}$ and $\text{COP} = 5.00$, then $\boxed{W = 24.0 \text{ J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h = Q_c + W = 120 \text{ J} + 24 \text{ J} = \boxed{144 \text{ J}}$$

P22.7 $\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_c}{t} = (0.0300 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(22.0^\circ\text{C}) + (0.0300 \text{ kg})(3.33 \times 10^5 \text{ J/kg}) \\ + (0.0300 \text{ kg})(2090 \text{ J/kg}^\circ\text{C})(20.0^\circ\text{C}) = 1.40 \times 10^4 \text{ J/min}$$

or, $\frac{Q_c}{t} = 233 \text{ J/s}$

Thus, the work done per second is

$$\mathcal{P} = \frac{233 \text{ J/s}}{3.00} = \boxed{77.8 \text{ W}}$$

P22.8 (a) $\left(10.0 \frac{\text{Btu}}{\text{h} \cdot \text{W}}\right) \left(\frac{1055 \text{ J}}{1 \text{ Btu}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{1 \text{ W}}{1 \text{ J/s}}\right) = \boxed{2.93}$

(b) The energy extracted by heat from the cold side divided by required work input is by definition the coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10000 \text{ Btu/h}}{\mathcal{P}}$; $\mathcal{P} = \frac{10000 \text{ Btu/h}}{5 \text{ Btu/h} \cdot \text{W}} = 2000 \text{ W} = 2.00 \text{ kW}$

Energy purchased is $\mathcal{P} \Delta t = (2.00 \text{ kW})(1500 \text{ h}) = 3.00 \times 10^3 \text{ kWh}$

Cost = $(3.00 \times 10^3 \text{ kWh})(0.100 \text{ \$/kWh}) = \$300$

With EER 10, $10 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10000 \text{ Btu/h}}{\mathcal{P}}$; $\mathcal{P} = \frac{10000 \text{ Btu/h}}{10 \text{ Btu/h} \cdot \text{W}} = 1000 \text{ W} = 1.00 \text{ kW}$

Energy purchased is $\mathcal{P} \Delta t = (1.00 \text{ kW})(1500 \text{ h}) = 1.50 \times 10^3 \text{ kWh}$

Cost = $(1.50 \times 10^3 \text{ kWh})(0.100 \text{ \$/kWh}) = \$150$

Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

Section 22.3 Reversible and Irreversible Processes

Section 22.4 The Carnot Engine

P22.9 $T_c = 703 \text{ K}$ $T_h = 2143 \text{ K}$

(a) $e_c = \frac{\Delta T}{T_h} = \frac{1440}{2143} = \boxed{67.2\%}$

(b) $|Q_h| = 1.40 \times 10^5 \text{ J}$, $W_{\text{eng}} = 0.420|Q_h|$

$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88 \times 10^4 \text{ J}}{1 \text{ s}} = \boxed{58.8 \text{ kW}}$$

P22.10 When $e = e_c$, $1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|}$ and $\frac{W_{\text{eng}}/\Delta t}{|Q_h|/\Delta t} = 1 - \frac{T_c}{T_h}$

(a) $|Q_h| = \frac{(W_{\text{eng}}/\Delta t)\Delta t}{1 - (T_c/T_h)} = \frac{(1.50 \times 10^5 \text{ W})(3600 \text{ s})}{1 - 293/773}$

$|Q_h| = 8.70 \times 10^8 \text{ J} = \boxed{870 \text{ MJ}}$

(b) $|Q_c| = |Q_h| - \left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t = 8.70 \times 10^8 - (1.50 \times 10^5)(3600) = 3.30 \times 10^8 \text{ J} = \boxed{330 \text{ MJ}}$

***P22.11** We use amounts of energy to find the actual efficiency. $Q_h = Q_c + W_{\text{eng}} = 20 \text{ kJ} + 1.5 \text{ kJ} = 21.5 \text{ kJ}$
 $e = W_{\text{eng}}/Q_h = 1.5 \text{ kJ}/21.5 \text{ kJ} = 0.0698$

We use temperatures to find the Carnot efficiency of a reversible engine $e_c = 1 - T_c/T_h = 1 - 373 \text{ K}/453 \text{ K} = 0.177$

The actual efficiency of 0.0698 is less than four-tenths of the Carnot efficiency of 0.177.

***P22.12** (a) $e_c = 1 - T_c/T_h = 1 - 350/500 = \boxed{0.300}$

(b) In $e_c = 1 - T_c/T_h$ we differentiate to find $de_c/dT_h = 0 - T_c(-1)T_h^{-2} = T_c/T_h^2 = 350/500^2 = \boxed{1.40 \times 10^{-3}}$ This is the increase of efficiency per degree of increase in the temperature of the hot reservoir.

(c) In $e_c = 1 - T_c/T_h$ we differentiate to find $de_c/dT_c = 0 - 1/T_h = -1/500 = -2.00 \times 10^{-3}$
 Then $de_c/(-dT_c) = \boxed{+2.00 \times 10^{-3}}$ This is the increase of efficiency per degree of decrease in the temperature of the cold reservoir. Note that it is a better deal to cool the exhaust than to supercharge the firebox.

P22.13 Isothermal expansion at $T_h = 523 \text{ K}$

Isothermal compression at $T_c = 323 \text{ K}$

Gas absorbs 1200 J during expansion.

(a) $|Q_c| = |Q_h| \left(\frac{T_c}{T_h}\right) = 1200 \text{ J} \left(\frac{323}{523}\right) = \boxed{741 \text{ J}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = (1200 - 741) \text{ J} = \boxed{459 \text{ J}}$

P22.14 The Carnot summer efficiency is $e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 20) \text{ K}}{(273 + 350) \text{ K}} = 0.530$

And in winter, $e_{c,w} = 1 - \frac{283}{623} = 0.546$

Then the actual winter efficiency is $0.320 \left(\frac{0.546}{0.530}\right) = \boxed{0.330}$ or $\boxed{33.0\%}$

P22.15 (a) In an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma$. Also, $\left(\frac{P_f V_f}{T_f}\right)^\gamma = \left(\frac{P_i V_i}{T_i}\right)^\gamma$

Dividing the second equation by the first yields $T_f = T_i \left(\frac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$

Since $\gamma = \frac{5}{3}$ for Argon, $\frac{\gamma-1}{\gamma} = \frac{2}{5} = 0.400$ and we have

$$T_f = (1073 \text{ K}) \left(\frac{300 \times 10^3 \text{ Pa}}{1.50 \times 10^6 \text{ Pa}}\right)^{0.400} = \boxed{564 \text{ K}}$$

(b) $\Delta E_{\text{int}} = nC_V \Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V \Delta T$,
and the power output is

$$\begin{aligned} \mathcal{P} &= \frac{W_{\text{eng}}}{t} = \frac{-nC_V \Delta T}{t} \text{ or} \\ &= \frac{(-80.0 \text{ kg})(1 \text{ mol}/0.0399 \text{ kg})\left(\frac{3}{2}\right)(8.314 \text{ J/mol} \cdot \text{K})(564 - 1073) \text{ K}}{60.0 \text{ s}} \end{aligned}$$

$$\mathcal{P} = 2.12 \times 10^5 \text{ W} = \boxed{212 \text{ kW}}$$

(c) $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{564 \text{ K}}{1073 \text{ K}} = 0.475$ or $\boxed{47.5\%}$

P22.16 (a) $e_{\text{max}} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12 \times 10^{-2} = \boxed{5.12\%}$

(b) $\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = 75.0 \times 10^6 \text{ J/s}$

Therefore, $W_{\text{eng}} = (75.0 \times 10^6 \text{ J/s})(3600 \text{ s/h}) = 2.70 \times 10^{11} \text{ J/h}$

From $e = \frac{W_{\text{eng}}}{|Q_h|}$ we find $|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70 \times 10^{11} \text{ J/h}}{5.12 \times 10^{-2}} = 5.27 \times 10^{12} \text{ J/h} = \boxed{5.27 \text{ TJ/h}}$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P22.17** (a) $e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{e_1 Q_{1h} + e_2 Q_{2h}}{Q_{1h}}$

Now

$$Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{1h} - e_1 Q_{1h}$$

So

$$e = \frac{e_1 Q_{1h} + e_2 (Q_{1h} - e_1 Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1 e_2}$$

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$$\begin{aligned}
 \text{(b)} \quad e &= e_1 + e_2 - e_1 e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right) \left(1 - \frac{T_c}{T_i}\right) \\
 &= 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}
 \end{aligned}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted.

$$\text{(c)} \quad \text{With } W_{\text{eng2}} = W_{\text{eng1}}, \quad e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$$

$$1 - \frac{T_c}{T_h} = 2 \left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}(T_h + T_c)}$$

$$\text{(d)} \quad e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = (T_h T_c)^{1/2}}$$

***P22.18** (a) “The actual efficiency is two thirds the Carnot efficiency” reads as an equation

$$\frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{|Q_c| + W_{\text{eng}}} = \frac{2}{3} \left(1 - \frac{T_c}{T_h}\right) = \frac{2}{3} \frac{T_h - T_c}{T_h}$$

All the T 's represent absolute temperatures. Then

$$\frac{|Q_c| + W_{\text{eng}}}{W_{\text{eng}}} = \frac{1.5 T_h}{T_h - T_c} \quad \frac{|Q_c|}{W_{\text{eng}}} = \frac{1.5 T_h}{T_h - T_c} - 1 = \frac{1.5 T_h - T_h + T_c}{T_h - T_c}$$

$$|Q_c| = W_{\text{eng}} \frac{0.5 T_h + T_c}{T_h - T_c} \quad \frac{|Q_c|}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t} \frac{0.5 T_h + T_c}{T_h - T_c} = \boxed{1.40 \text{ MW} \frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}}}$$

The dominating T_h in the bottom of this fraction means that the exhaust power decreases as the firebox temperature increases.

$$\text{(b)} \quad \frac{|Q_c|}{\Delta t} = 1.40 \text{ MW} \frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} = 1.40 \text{ MW} \frac{0.5(1073 \text{ K}) + 383 \text{ K}}{(1073 - 383) \text{ K}} = \boxed{1.87 \text{ MW}}$$

$$\text{(c)} \quad \text{We require } \frac{|Q_c|}{\Delta t} = \frac{1}{2} 1.87 \text{ MW} = 1.40 \text{ MW} \frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} \quad \frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} = 0.666$$

$$0.5 T_h + 383 \text{ K} = 0.666 T_h - 255 \text{ K} \quad T_h = 638 \text{ K} / 0.166 = \boxed{3.84 \times 10^3 \text{ K}}$$

(d) The minimum possible heat exhaust power is approached as the firebox temperature goes to infinity, and it is $|Q_c|/\Delta t = 1.40 \text{ MW} (0.5/1) = 0.7 \text{ MW}$. The heat exhaust power cannot be as small as $(1/4)(1.87 \text{ MW}) = 0.466 \text{ MW}$. So

no answer exists. The energy exhaust cannot be that small.

$$\mathbf{P22.19} \quad (\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270}{30.0} = \boxed{9.00}$$

P22.20 (a) First, consider the adiabatic process $D \rightarrow A$:

$$P_D V_D^\gamma = P_A V_A^\gamma \quad \text{so} \quad P_D = P_A \left(\frac{V_A}{V_D} \right)^\gamma = 1\,400 \text{ kPa} \left(\frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^{5/3} = \boxed{712 \text{ kPa}}$$

$$\text{Also} \quad \left(\frac{nRT_D}{V_D} \right) V_D^\gamma = \left(\frac{nRT_A}{V_A} \right) V_A^\gamma$$

$$\text{or} \quad T_D = T_A \left(\frac{V_A}{V_D} \right)^{\gamma-1} = 720 \text{ K} \left(\frac{10.0}{15.0} \right)^{2/3} = \boxed{549 \text{ K}}$$

Now, consider the isothermal process $C \rightarrow D$: $T_C = T_D = \boxed{549 \text{ K}}$

$$P_C = P_D \left(\frac{V_D}{V_C} \right) = \left[P_A \left(\frac{V_A}{V_D} \right)^\gamma \right] \left(\frac{V_D}{V_C} \right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400 \text{ kPa} (10.0 \text{ L})^{5/3}}{24.0 \text{ L} (15.0 \text{ L})^{2/3}} = \boxed{445 \text{ kPa}}$$

Next, consider the adiabatic process $B \rightarrow C$: $P_B V_B^\gamma = P_C V_C^\gamma$

But, $P_C = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A \left(\frac{V_A}{V_B} \right)$

$$\text{Hence, } P_A \left(\frac{V_A}{V_B} \right) V_B^\gamma = \left(\frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}} \right) V_C^\gamma \text{ which reduces to } V_B = \frac{V_A V_C}{V_D} = \frac{10.0 \text{ L} (24.0 \text{ L})}{15.0 \text{ L}} = \boxed{16.0 \text{ L}}$$

$$\text{Finally, } P_B = P_A \left(\frac{V_A}{V_B} \right) = 1\,400 \text{ kPa} \left(\frac{10.0 \text{ L}}{16.0 \text{ L}} \right) = \boxed{875 \text{ kPa}}$$

State	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

continued on next page

(b) For the isothermal process $A \rightarrow B$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

so $Q = -W = nRT \ln\left(\frac{V_B}{V_A}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(720 \text{ K}) \ln\left(\frac{16.0}{10.0}\right) = \boxed{+6.58 \text{ kJ}}$

For the adiabatic process $B \rightarrow C$: $Q = \boxed{0}$

$\Delta E_{\text{int}} = nC_V(T_C - T_B) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](549 - 720) \text{ K} = \boxed{-4.98 \text{ kJ}}$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98 \text{ kJ}) = \boxed{-4.98 \text{ kJ}}$

For the isothermal process $C \rightarrow D$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

and $Q = -W = nRT \ln\left(\frac{V_D}{V_C}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(549 \text{ K}) \ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02 \text{ kJ}}$

Finally, for the adiabatic process $D \rightarrow A$: $Q = \boxed{0}$

$\Delta E_{\text{int}} = nC_V(T_A - T_D) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](720 - 549) \text{ K} = \boxed{+4.98 \text{ kJ}}$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = \boxed{+4.98 \text{ kJ}}$

Process	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \rightarrow B$	+6.58	-6.58	0
$B \rightarrow C$	0	-4.98	-4.98
$C \rightarrow D$	-5.02	+5.02	0
$D \rightarrow A$	0	+4.98	+4.98
$ABCD A$	+1.56	-1.56	0

The work done by the engine is the negative of the work input. The output work W_{eng} is given by the work column in the table with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A \rightarrow B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237$ or $\boxed{23.7\%}$

P22.21 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c| \left[\frac{|Q_h|}{|Q_c|} - 1 \right]$

The text shows that for a Carnot cycle (and only for a reversible cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$

Therefore,

$$W = |Q_c| \left[\frac{T_h - T_c}{T_c} \right]$$

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$

Using the result from part (a), this becomes $\text{COP} = \frac{T_c}{T_h - T_c}$

$$\text{P22.22} \quad (\text{COP})_{\text{heat pump}} = \frac{|Q_c| + W}{W} = \frac{T_h}{\Delta T} = \frac{295}{25} = \boxed{11.8}$$

$$\text{P22.23} \quad (\text{COP})_{\text{Carnot refriger}} = \frac{T_c}{\Delta T} = \frac{4.00}{289} = 0.0138 = \frac{|Q_c|}{W}$$

$\therefore W = \boxed{72.2 \text{ J}}$ per 1 J energy removed by heat.

$$\text{P22.24} \quad \text{COP} = 0.100 \text{COP}_{\text{Carnot cycle}}$$

or

$$\frac{|Q_h|}{W} = 0.100 \left(\frac{|Q_h|}{W} \right)_{\text{Carnot cycle}} = 0.100 \left(\frac{1}{\text{Carnot efficiency}} \right)$$

$$\frac{|Q_h|}{W} = 0.100 \left(\frac{T_h}{T_h - T_c} \right) = 0.100 \left(\frac{293 \text{ K}}{293 \text{ K} - 268 \text{ K}} \right) = 1.17$$

Thus, $\boxed{1.17 \text{ joules of energy enter the room by heat for each joule of work done.}}$

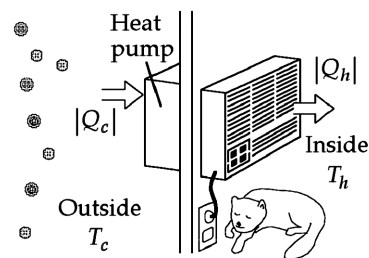


FIG. P22.24

$$\text{*P22.25} \quad \frac{|Q_c|}{W} = \text{COP}_c (\text{refrigerator}) = \frac{T_c}{T_h - T_c} = \frac{|Q_c|/\Delta t}{W/\Delta t}$$

$$\frac{0.150 \text{ W}}{W/\Delta t} = \frac{260 \text{ K}}{40.0 \text{ K}}$$

$$\mathcal{P} = \frac{W}{\Delta t} = 0.150 \text{ W} \left(\frac{40.0 \text{ K}}{260 \text{ K}} \right) = \boxed{23.1 \text{ mW}}$$

$$\text{P22.26} \quad e = \frac{W}{Q_h} = 0.350 \quad W = 0.350 Q_h$$

$$Q_h = W + Q_c \quad Q_c = 0.650 Q_h$$

$$\text{COP}(\text{refrigerator}) = \frac{Q_c}{W} = \frac{0.650 Q_h}{0.350 Q_h} = \boxed{1.86}$$

Section 22.5 Gasoline and Diesel Engines

$$\text{P22.27} \quad (\text{a}) \quad P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = (3.00 \times 10^6 \text{ Pa}) \left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3} \right)^{1.40} = \boxed{244 \text{ kPa}}$$

$$(\text{b}) \quad W = \int_{V_i}^{V_f} P dV \quad P = P_i \left(\frac{V_i}{V} \right)^\gamma$$

Integrating,

$$\begin{aligned} W &= \left(\frac{1}{\gamma - 1} \right) P_i V_i \left[1 - \left(\frac{V_i}{V_f} \right)^{\gamma - 1} \right] \\ &= (2.50)(3.00 \times 10^6 \text{ Pa})(5.00 \times 10^{-5} \text{ m}^3) \left[1 - \left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3} \right)^{0.400} \right] \\ &= \boxed{192 \text{ J}} \end{aligned}$$

P22.28 (a), (b) The quantity of gas is

$$n = \frac{P_A V_A}{RT_A} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.0205 \text{ mol}$$

$$E_{\text{int},A} = \frac{5}{2} nRT_A = \frac{5}{2} P_A V_A = \frac{5}{2} (100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3) = \boxed{125 \text{ J}}$$

$$\text{In process } AB, P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma = (100 \times 10^3 \text{ Pa})(8.00)^{1.40} = \boxed{1.84 \times 10^6 \text{ Pa}}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{(1.84 \times 10^6 \text{ Pa})(500 \times 10^{-6} \text{ m}^3 / 8.00)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = \boxed{673 \text{ K}}$$

$$E_{\text{int},B} = \frac{5}{2} nRT_B = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(673 \text{ K}) = \boxed{287 \text{ J}}$$

$$\text{so } \Delta E_{\text{int},AB} = 287 \text{ J} - 125 \text{ J} = \boxed{162 \text{ J}} = Q - W_{\text{out}} = 0 - W_{\text{out}} \quad W_{AB} = \boxed{-162 \text{ J}}$$

Process BC takes us to:

$$P_C = \frac{nRT_C}{V_C} = \frac{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K})}{62.5 \times 10^{-6} \text{ m}^3} = \boxed{2.79 \times 10^6 \text{ Pa}}$$

$$E_{\text{int},C} = \frac{5}{2} nRT_C = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K}) = \boxed{436 \text{ J}}$$

$$E_{\text{int},BC} = 436 \text{ J} - 287 \text{ J} = \boxed{149 \text{ J}} = Q - W_{\text{out}} = Q - 0$$

$$Q_{BC} = \boxed{149 \text{ J}}$$

In process CD :

$$P_D = P_C \left(\frac{V_C}{V_D} \right)^\gamma = (2.79 \times 10^6 \text{ Pa}) \left(\frac{1}{8.00} \right)^{1.40} = \boxed{1.52 \times 10^5 \text{ Pa}}$$

$$T_D = \frac{P_D V_D}{nR} = \frac{(1.52 \times 10^5 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = \boxed{445 \text{ K}}$$

$$E_{\text{int},D} = \frac{5}{2} nRT_D = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(445 \text{ K}) = \boxed{190 \text{ J}}$$

$$\Delta E_{\text{int},CD} = 190 \text{ J} - 436 \text{ J} = \boxed{-246 \text{ J}} = Q - W_{\text{out}} = 0 - W_{\text{out}}$$

$$W_{CD} = \boxed{246 \text{ J}}$$

$$\text{and } \Delta E_{\text{int},DA} = E_{\text{int},A} - E_{\text{int},D} = 125 \text{ J} - 190 \text{ J} = \boxed{-65.0 \text{ J}} = Q - W_{\text{out}} = Q - 0$$

$$Q_{DA} = \boxed{-65.0 \text{ J}}$$

For the entire cycle, $\Delta E_{\text{int},\text{net}} = 162 \text{ J} + 149 - 246 - 65.0 = \boxed{0}$. The net work is

$$W_{\text{eng}} = -162 \text{ J} + 0 + 246 \text{ J} + 0 = \boxed{84.3 \text{ J}}$$

$$Q_{\text{net}} = 0 + 149 \text{ J} + 0 - 65.0 \text{ J} = \boxed{84.3 \text{ J}}$$

continued on next page

The tables look like:

State	T (K)	P (kPa)	V (cm ³)	E_{int} (J)
A	293	100	500	125
B	673	1840	62.5	287
C	1023	2790	62.5	436
D	445	152	500	190
A	293	100	500	125

Process	Q (J)	output W (J)	ΔE_{int} (J)
AB	0	-162	162
BC	149	0	149
CD	0	246	-246
DA	-65.0	0	-65.0
ABCD	84.3	84.3	0

(c) The input energy is $Q_h = 149 \text{ J}$, the waste is $|Q_c| = 65.0 \text{ J}$, and $W_{\text{eng}} = 84.3 \text{ J}$.

(d) The efficiency is: $e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3 \text{ J}}{149 \text{ J}} = 0.565$

(e) Let f represent the angular speed of the crankshaft. Then $\frac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1000 \text{ J/s} = \left(\frac{f}{2}\right)(84.3 \text{ J/cycle})$$

$$f = \frac{2000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = 1.42 \times 10^3 \text{ rev/min}$$

P22.29 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$$e = 1 - \left(\frac{1}{6.00}\right)^{0.400} = 51.2\%$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = 36.2\%$

Section 22.6 Entropy

P22.30 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500 \text{ kg})(3.33 \times 10^5 \text{ J/kg})}{273 \text{ K}} = -610 \text{ J/K}$$

***P22.31** The process of raising the temperature of the sample in this way is reversible, because an infinitesimal change would make δ negative, and energy would flow out instead of in. Then we may find the entropy change of the sample as

$$\Delta S = \int_{T_i}^{T_f} dS = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mcdT}{T} = mc \ln T \Big|_{T_i}^{T_f} = mc [\ln T_f - \ln T_i] = mc \ln(T_f/T_i)$$

***P22.32** (a) The process is isobaric because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (because energy goes in by heat), isothermal (T goes up), isovolumetric (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.

(b) The final temperature is

$$220^\circ\text{F} = 212^\circ\text{F} + 8^\circ\text{F} = 100^\circ\text{C} + 8^\circ\text{F} \left(\frac{100 - 0^\circ\text{C}}{212 - 32^\circ\text{F}} \right) = 104^\circ\text{C}$$

For the mixture,

$$\begin{aligned} Q &= m_1 c_1 \Delta T + m_2 c_2 \Delta T \\ &= (900 \text{ g } 1 \text{ cal/g} \cdot ^\circ\text{C} + 930 \text{ g } 0.299 \text{ cal/g} \cdot ^\circ\text{C})(104.4^\circ\text{C} - 23^\circ\text{C}) \\ &= 9.59 \times 10^4 \text{ cal} = \boxed{4.02 \times 10^5 \text{ J}} \end{aligned}$$

(c) Consider the reversible heating process described in part (a):

$$\begin{aligned} \Delta S &= \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1 c_1 + m_2 c_2) dT}{T} = (m_1 c_1 + m_2 c_2) \ln \frac{T_f}{T_i} \\ &= [900(1) + 930(0.299)] (\text{cal}/^\circ\text{C}) \left(\frac{4.186 \text{ J}}{1 \text{ cal}} \right) \left(\frac{1^\circ\text{C}}{1 \text{ K}} \right) \ln \left(\frac{273 + 104}{273 + 23} \right) \\ &= (4930 \text{ J/K}) 0.243 = \boxed{1.20 \times 10^3 \text{ J/K}} \end{aligned}$$

P22.33
$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mcdT}{T} = mc \ln \left(\frac{T_f}{T_i} \right)$$

$$\Delta S = 250 \text{ g} (1.00 \text{ cal/g} \cdot ^\circ\text{C}) \ln \left(\frac{353}{293} \right) = 46.6 \text{ cal/K} = \boxed{195 \text{ J/K}}$$

Section 22.7 Entropy Changes in Irreversible Processes

P22.34
$$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1000}{290} - \frac{1000}{5700} \right) \text{ J/K} = \boxed{3.27 \text{ J/K}}$$

P22.35 The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293} \text{ J/K} = \boxed{1.02 \text{ kJ/K}}$$

***P22.36** Define $T_1 = \text{Temp Cream} = 5.00^\circ\text{C} = 278 \text{ K}$. Define $T_2 = \text{Temp Coffee} = 60.0^\circ\text{C} = 333 \text{ K}$

The final temperature of the mixture is: $T_f = \frac{(20.0 \text{ g})T_1 + (200 \text{ g})T_2}{220 \text{ g}} = 55.0^\circ\text{C} = 328 \text{ K}$

The entropy change due to this mixing is $\Delta S = (20.0 \text{ g}) \int_{T_1}^{T_f} \frac{c_v dT}{T} + (200 \text{ g}) \int_{T_2}^{T_f} \frac{c_v dT}{T}$

$$\Delta S = (84.0 \text{ J/K}) \ln\left(\frac{T_f}{T_1}\right) + (840 \text{ J/K}) \ln\left(\frac{T_f}{T_2}\right) = (84.0 \text{ J/K}) \ln\left(\frac{328}{278}\right) + (840 \text{ J/K}) \ln\left(\frac{328}{333}\right)$$

$$\Delta S = \boxed{+1.18 \text{ J/K}}$$

P22.37 Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2500 \text{ kcal/d} = \frac{2500 \times 10^3 \text{ cal}}{86400 \text{ s}} \left(\frac{4.186 \text{ J}}{1 \text{ cal}} \right) = 120 \text{ W}$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120 \text{ W}}{293 \text{ K}} = 0.4 \text{ W/K} \sim \boxed{1 \text{ W/K}}$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P22.38 $c_{\text{iron}} = 448 \text{ J/kg}\cdot^\circ\text{C}$; $c_{\text{water}} = 4186 \text{ J/kg}\cdot^\circ\text{C}$

$$Q_{\text{cold}} = -Q_{\text{hot}}: \quad 4.00 \text{ kg}(4186 \text{ J/kg}\cdot^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00 \text{ kg})(448 \text{ J/kg}\cdot^\circ\text{C})(T_f - 900^\circ\text{C})$$

which yields $T_f = 33.2^\circ\text{C} = 306.2 \text{ K}$

$$\Delta S = \int_{283 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1173}\right)$$

$$\Delta S = (4186 \text{ J/kg}\cdot\text{K})(4.00 \text{ kg})(0.0788) + (448 \text{ J/kg}\cdot\text{K})(1.00 \text{ kg})(-1.34)$$

$$\Delta S = \boxed{718 \text{ J/K}}$$

P22.39 $\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = R \ln 2 = \boxed{5.76 \text{ J/K}}$

There is $\boxed{\text{no change in temperature}}$ for an ideal gas.

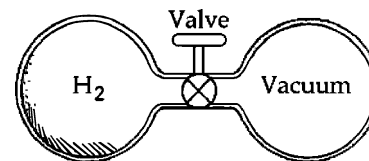


FIG. P22.39

P22.40 $\Delta S = nR \ln \left(\frac{V_f}{V_i} \right) = (0.044 \text{ mol})(2)R \ln 2$

$$\Delta S = 0.088 \text{ mol}(8.314 \text{ J/K}) \ln 2 = \boxed{0.507 \text{ J/K}}$$

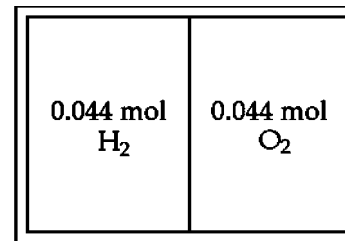


FIG. P22.40

Section 22.8 Entropy on a Microscopic Scale

P22.41 (a) A 12 can only be obtained way, as 6 + 6

(b) A 7 can be obtained ways: 6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6

P22.42 (a) The table is shown below. On the basis of the table, the most probable recorded result of a toss is .

(b) The most ordered state is the least likely macrostate. Thus, on the basis of the table this is .

(c) The most disordered is the most likely macrostate. Thus, this is .

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

P22.43 (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGR, RRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RRGR, GRRRG, RRGG, RGRGR, GRRGR, RGGRR, GRGR, GGRR	10
2R, 3G	GGRR, GGRGR, GRGG, RGGGR, GGRRG, GRGRG, RGGGR, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGR	5
All green	GGGGG	1

Additional Problems

P22.44 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount $mg y$, to the water from a stove at a temperature infinitesimally above 20.0°C . Then,

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{mgy}{T} = \frac{5\,000\text{ m}^3 (1\,000\text{ kg/m}^3)(9.80\text{ m/s}^2)(50.0\text{ m})}{293\text{ K}} = \boxed{8.36 \times 10^6\text{ J/K}}$$

***P22.45** For the Carnot engine, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.600$

Also,
$$e_c = \frac{W_{\text{eng}}}{|Q_h|}$$

so
$$|Q_h| = \frac{W_{\text{eng}}}{e_c} = \frac{150\text{ J}}{0.600} = 250\text{ J}$$

and
$$|Q_c| = |Q_h| - W_{\text{eng}} = 250\text{ J} - 150\text{ J} = 100\text{ J}$$

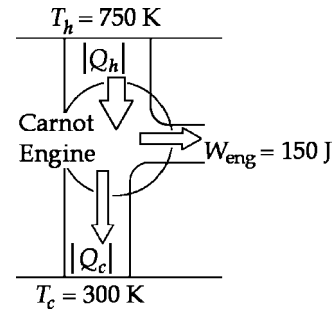


FIG. P22.45

(a) $|Q_h| = \frac{W_{\text{eng}}}{e_s} = \frac{150\text{ J}}{0.700} = \boxed{214\text{ J}}$

$$|Q_c| = |Q_h| - W_{\text{eng}} = 214\text{ J} - 150\text{ J} = \boxed{64.3\text{ J}}$$

(b) $|Q_{h,\text{net}}| = 214\text{ J} - 250\text{ J} = \boxed{-35.7\text{ J}}$

$$|Q_{c,\text{net}}| = 64.3\text{ J} - 100\text{ J} = \boxed{-35.7\text{ J}}$$

The net flow of energy by heat from the cold to the hot reservoir without work input, is impossible.

(c) For engine S: $|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_s} - W_{\text{eng}}$

so
$$W_{\text{eng}} = \frac{|Q_c|}{\frac{1}{e_s} - 1} = \frac{100\text{ J}}{\frac{1}{0.700} - 1} = \boxed{233\text{ J}}$$

and
$$|Q_h| = |Q_c| + W_{\text{eng}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$$

(d) $|Q_{h,\text{net}}| = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$

$$W_{\text{net}} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$$

$$|Q_{c,\text{net}}| = \boxed{0}$$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

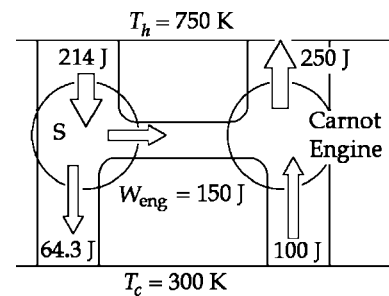


FIG. P22.45(b)

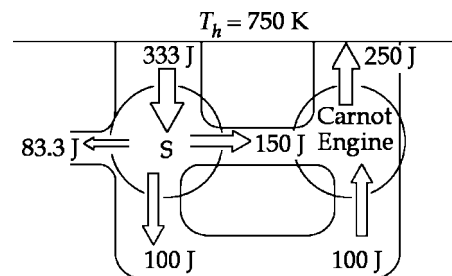


FIG. P22.45(d)

continued on next page

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$

Thus,

$$\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3 \text{ J}}{750 \text{ K}} + \frac{0}{300 \text{ K}} = \boxed{-0.111 \text{ J/K}}$$

A decrease in total entropy is impossible.

***P22.46** (a) Let state i represent the gas before its compression and state f afterwards, $V_f = \frac{V_i}{8}$. For a

diatomic ideal gas, $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, and $\gamma = \frac{C_p}{C_v} = 1.40$. Next,

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = P_i 8^{1.40} = 18.4 P_i$$

$$P_i V_i = nRT_i$$

$$P_f V_f = \frac{18.4 P_i V_i}{8} = 2.30 P_i V_i = 2.30 nRT_i = nRT_f$$

so $T_f = 2.30 T_i$

$$\begin{aligned} \Delta E_{\text{int}} &= nC_v \Delta T = n \frac{5}{2} R (T_f - T_i) = \frac{5}{2} nR (1.30 T_i - T_i) = \frac{5}{2} (1.30 - 1) nRT_i \\ &= \frac{5}{2} (1.30 - 1) (1.013 \times 10^5 \text{ N/m}^2) (0.12 \times 10^{-3} \text{ m}^3) = 39.4 \text{ J} \end{aligned}$$

Since the process is adiabatic, $Q = 0$ and $\Delta E_{\text{int}} = Q + W$ gives $W = \boxed{39.4 \text{ J}}$

(b) The moment of inertia of the wheel is $I = \frac{1}{2} MR^2 = \frac{1}{2} (5.1 \text{ kg})(0.085 \text{ m})^2 = 0.0184 \text{ kg} \cdot \text{m}^2$

We want the flywheel to do work 39.4 J, so the work on the flywheel should be -39.4 J :

$$K_{\text{rot } i} + W = K_{\text{rot } f}$$

$$\frac{1}{2} I \omega_i^2 - 39.4 \text{ J} = 0$$

$$\omega_i = \left(\frac{2(39.4 \text{ J})}{0.0184 \text{ kg} \cdot \text{m}^2} \right)^{1/2} = \boxed{65.4 \text{ rad/s}}$$

(c) Now we want $W = 0.05 K_{\text{rot } i}$

$$39.4 \text{ J} = 0.05 \frac{1}{2} (0.0184 \text{ kg} \cdot \text{m}^2) \omega_i^2$$

$$\omega = \left(\frac{2(789 \text{ J})}{0.0184 \text{ kg} \cdot \text{m}^2} \right)^{1/2} = \boxed{293 \text{ rad/s}}$$

- P22.47** (a) $\mathcal{P}_{\text{electric}} = \frac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore,

$$\mathcal{P}_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5\,000 \text{ W}}$$

- (b) For a heat pump, $(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295 \text{ K}}{27 \text{ K}} = 10.92$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathcal{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000 \text{ W}}{6.56} = \boxed{763 \text{ W}}$$

P22.48 $\Delta S_{\text{hot}} = \frac{-1\,000 \text{ J}}{600 \text{ K}}$

$$\Delta S_{\text{cold}} = \frac{+750 \text{ J}}{350 \text{ K}}$$

(a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476 \text{ J/K}}$

(b) $e_c = 1 - \frac{T_1}{T_2} = 0.417$

$$W_{\text{eng}} = e_c |Q_h| = 0.417(1\,000 \text{ J}) = \boxed{417 \text{ J}}$$

(c) $W_{\text{net}} = 417 \text{ J} - 250 \text{ J} = 167 \text{ J}$

$$T_1 \Delta S_U = 350 \text{ K}(0.476 \text{ J/K}) = \boxed{167 \text{ J}}$$

- P22.49** (a) For an isothermal process, $Q = nRT \ln\left(\frac{V_2}{V_1}\right)$

Therefore,

$$Q_1 = nR(3T_i) \ln 2$$

and

$$Q_3 = nR(T_i) \ln\left(\frac{1}{2}\right)$$

For the constant volume processes, $Q_2 = \Delta E_{\text{int},2} = \frac{3}{2} nR(T_i - 3T_i)$

and

$$Q_4 = \Delta E_{\text{int},4} = \frac{3}{2} nR(3T_i - T_i)$$

The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or

$$Q = \boxed{2nRT_i \ln 2}$$

- (b) A positive value for heat represents energy transferred into the system.

Therefore,

$$|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

Therefore, the efficiency is

$$e_c = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2 \ln 2}{3(1 + \ln 2)} = \boxed{0.273}$$

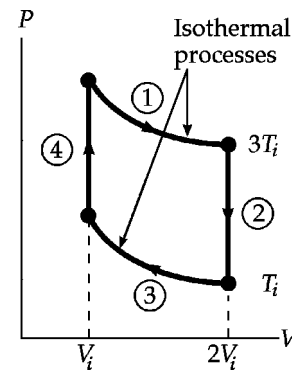


FIG. P22.49

P22.50 (a) $35.0^\circ\text{F} = \frac{5}{9}(35.0 - 32.0)^\circ\text{C} = (1.67 + 273.15) \text{ K} = 274.82 \text{ K}$

$$98.6^\circ\text{F} = \frac{5}{9}(98.6 - 32.0)^\circ\text{C} = (37.0 + 273.15) \text{ K} = 310.15 \text{ K}$$

$$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6 \text{ g})(1.00 \text{ cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T} = 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86 \text{ cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00) \frac{(310.15 - 274.82)}{310.15} = -51.67 \text{ cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19 \text{ cal/K}}$$

(b) $(453.6)(1)(T_F - 274.82) = (70.0 \times 10^3)(1)(310.15 - T_F)$

Thus,

$$(70.0 + 0.4536) \times 10^3 T_F = [(70.0)(310.15) + (0.4536)(274.82)] \times 10^3$$

$$\text{and } T_F = 309.92 \text{ K} = 36.77^\circ\text{C} = \boxed{98.19^\circ\text{F}}$$

$$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52 \text{ cal/K}$$

$$\Delta S'_{\text{body}} = -(70.0 \times 10^3) \ln\left(\frac{310.15}{309.92}\right) = -51.93 \text{ cal/K}$$

$$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59 \text{ cal/K}}$$

This is significantly less than the estimate in part (a).

P22.51 $e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}} / \Delta t}{|Q_h| / \Delta t}$: $\frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{(1 - T_c/T_h)} = \frac{\mathcal{P} T_h}{T_h - T_c}$

$$|Q_h| = W_{\text{eng}} + |Q_c|:$$

$$\frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{\text{eng}}}{\Delta t}$$

$$\frac{|Q_c|}{\Delta t} = \frac{\mathcal{P} T_h}{T_h - T_c} - \mathcal{P} = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$$|Q_c| = mc\Delta T:$$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) c\Delta T = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P} T_c}{(T_h - T_c) c\Delta T}$$

$$\frac{\Delta m}{\Delta t} = \frac{(1.00 \times 10^9 \text{ W})(300 \text{ K})}{200 \text{ K}(4186 \text{ J/kg}\cdot^\circ\text{C})(6.00^\circ\text{C})} = \boxed{5.97 \times 10^4 \text{ kg/s}}$$

$$\mathbf{P22.52} \quad e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}} / \Delta t}{|Q_h| / \Delta t} \quad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{1 - (T_c / T_h)} = \frac{\mathcal{P} T_h}{T_h - T_c}$$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t} \right) - \mathcal{P} = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where c is the specific heat of water.

Therefore,
$$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t} \right) c \Delta T = \frac{\mathcal{P} T_c}{T_h - T_c}$$

and
$$\frac{\Delta m}{\Delta t} = \boxed{\frac{\mathcal{P} T_c}{(T_h - T_c) c \Delta T}}$$

We test for dimensional correctness by identifying the units of the right-hand side:

$$\frac{\text{W} \cdot ^\circ\text{C}}{^\circ\text{C}(\text{J}/\text{kg} \cdot ^\circ\text{C})^\circ\text{C}} = \frac{(\text{J}/\text{s})\text{kg}}{\text{J}} = \text{kg}/\text{s}, \text{ as on the left hand side. Think of yourself as a power-company}$$

engineer arranging to have enough cooling water to carry off your thermal pollution. If the plant power \mathcal{P} increases, the required flow rate increases in direct proportion. If environmental regulations require a smaller temperature change ΔT , then the required flow rate increases again, now in inverse proportion. Next note that T_h is in the bottom of the fraction. This means that if you can run the reactor core or firebox hotter, the required coolant flow rate decreases! If the turbines take in steam at higher temperature, they can be made more efficient to reduce waste heat output.

P22.53 Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is
$$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280 \text{ K}}{20 \text{ K}} = 14.0$$

(a) Its actual COP is
$$0.400(14.0) = 5.60 = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$$

$$5.60 \left| \frac{Q_h}{\Delta t} \right| - 5.60 \left| \frac{Q_c}{\Delta t} \right| = \left| \frac{Q_c}{\Delta t} \right|$$

$$5.60(10.0 \text{ kW}) = 6.60 \left| \frac{Q_c}{\Delta t} \right| \quad \text{and} \quad \left| \frac{Q_c}{\Delta t} \right| = \boxed{8.48 \text{ kW}}$$

(b) $|Q_h| = W_{\text{eng}} + |Q_c|$:
$$\frac{W_{\text{eng}}}{\Delta t} = \left| \frac{Q_h}{\Delta t} \right| - \left| \frac{Q_c}{\Delta t} \right| = 10.0 \text{ kW} - 8.48 \text{ kW} = \boxed{1.52 \text{ kW}}$$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0 \times 10^3 \text{ J/s})(3600 \text{ s})}{300 \text{ K}} = 1.20 \times 10^5 \text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48 \times 10^3 \text{ J/s})(3600 \text{ s})}{280 \text{ K}} = -1.09 \times 10^5 \text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20 \times 10^5 \text{ J/K} - 1.09 \times 10^5 \text{ J/K} = \boxed{1.09 \times 10^4 \text{ J/K}}$$

(d) The new ideal COP is
$$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280 \text{ K}}{25 \text{ K}} = 11.2$$

We suppose the actual COP is $0.400(11.2) = 4.48$

As a fraction of the original 5.60, this is $\frac{4.48}{5.60} = 0.800$, so the fractional change is to

$\boxed{\text{drop by } 20.0\%}$.

*P22.54 (a) For the isothermal process AB , the work on the gas is

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5(1.013 \times 10^5 \text{ Pa})(10.0 \times 10^{-3} \text{ m}^3) \ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15 \times 10^3 \text{ J}$$

where we have used $1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

and $1.00 \text{ L} = 1.00 \times 10^{-3} \text{ m}^3$

$$W_{BC} = -P_B \Delta V = -(1.013 \times 10^5 \text{ Pa})[(10.0 - 50.0) \times 10^{-3}] \text{ m}^3 = +4.05 \times 10^3 \text{ J}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.10 \times 10^3 \text{ J} = \boxed{4.10 \text{ kJ}}$$

(b) Since AB is an isothermal process, $\Delta E_{\text{int}, AB} = 0$

$$\text{and } Q_{AB} = -W_{AB} = 8.15 \times 10^3 \text{ J}$$

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{(1.013 \times 10^5)(50.0 \times 10^{-3})}{R} = \frac{5.06 \times 10^3}{R}$$

$$\text{Also, } T_C = \frac{P_C V_C}{nR} = \frac{(1.013 \times 10^5)(10.0 \times 10^{-3})}{R} = \frac{1.01 \times 10^3}{R}$$

$$Q_{CA} = nC_V \Delta T = 1.00 \left(\frac{3}{2}R\right) \left(\frac{5.06 \times 10^3 - 1.01 \times 10^3}{R}\right) \\ = 6.08 \text{ kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15 \text{ kJ} + 6.08 \text{ kJ} = \boxed{14.2 \text{ kJ}}$

$$(c) Q_{BC} = nC_P \Delta T = \frac{5}{2}(nR \Delta T) = \frac{5}{2}P_B \Delta V_{BC}$$

$$Q_{BC} = \frac{5}{2}(1.013 \times 10^5)[(10.0 - 50.0) \times 10^{-3}] = -1.01 \times 10^4 \text{ J} = \boxed{-10.1 \text{ kJ}}$$

$$(d) e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB} + Q_{CA}} = \frac{4.10 \times 10^3 \text{ J}}{1.42 \times 10^4 \text{ J}} = 0.288 \text{ or } \boxed{28.8\%}$$

(e) A Carnot engine operating between $T_{\text{hot}} = T_A = 5060/R$ and $T_{\text{cold}} = T_C = 1010/R$ has efficiency $1 - T_c/T_h = 1 - 1/5 = 80.0\%$.

The three-process engine considered in this problem has much lower efficiency.

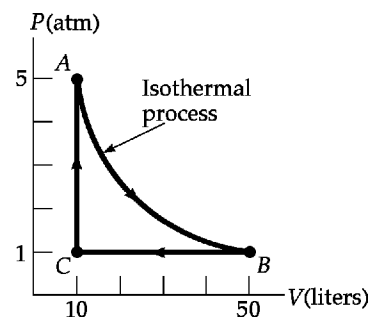


FIG. P22.54

*P22.55 At point A, $P_i V_i = nRT_i$ and $n = 1.00$ mol

At point B, $3P_i V_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$

and $C_P = \frac{5R}{2}$:

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$

$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$

$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$

$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{T_i}{6T_i} = \boxed{0.833}$

The Carnot efficiency is much higher.

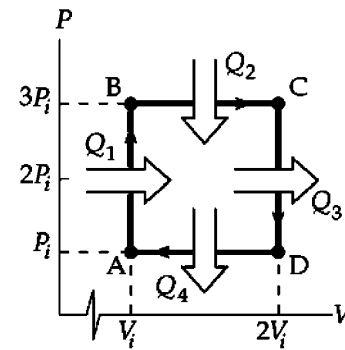


FIG. P22.55

P22.56
$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T \Big|_{T_i}^{T_f} = nC_P (\ln T_f - \ln T_i) = nC_P \ln \left(\frac{T_f}{T_i} \right)$$

$$\Delta S = nC_P \ln \left(\frac{PV_f}{nR} \frac{nR}{PV_i} \right) = \boxed{nC_P \ln 3}$$

- P22.57** (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, T_i represent the state of the gas before the isothermal expansion. Let P_c, V_c, T_c represent the state after this process, so that $P_i V_i = P_c V_c$. Let $P_f, 3V_i, T_f$ represent the state after the adiabatic compression.

$$\text{Then} \quad P_c V_c^\gamma = P_i (3V_i)^\gamma$$

$$\text{Substituting} \quad P_c = \frac{P_i V_i}{V_c}$$

$$\text{gives} \quad P_i V_i V_c^{\gamma-1} = P_i (3^\gamma V_i^\gamma)$$

$$\text{Then} \quad V_c^{\gamma-1} = 3^\gamma V_i^{\gamma-1} \quad \text{and} \quad \frac{V_c}{V_i} = 3^{\gamma/(\gamma-1)}$$

The work output in the isothermal expansion is

$$W = \int_i^c P dV = nRT_i \int_i^c V^{-1} dV = nRT_i \ln\left(\frac{V_c}{V_i}\right) = nRT_i \ln(3^{\gamma/(\gamma-1)}) = nRT_i \left(\frac{\gamma}{\gamma-1}\right) \ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S = \frac{Q}{T} = nR \left(\frac{\gamma}{\gamma-1}\right) \ln 3$$

$$\text{Since} \quad C_p = \gamma C_v = C_v + R$$

$$\text{we have} \quad (\gamma-1)C_v = R, \quad C_v = \frac{R}{\gamma-1}$$

$$\text{and} \quad C_p = \frac{\gamma R}{\gamma-1}$$

$$\text{Then the result is} \quad \boxed{\Delta S = nC_p \ln 3}$$

- (b) The pair of processes considered here carries the gas from the initial state in Problem 56 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 56 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to Problems 56 and 57(a) must be the same.

$$*\mathbf{P22.58} \quad (a) \quad W = \int_{V_i}^{V_f} P dV = nRT \int_{V_i}^{2V_i} \frac{dV}{V} = (1.00) RT \ln\left(\frac{2V_i}{V_i}\right) = \boxed{RT \ln 2}$$

- (b) While it lasts, this process does convert all of the energy input into work output. But the gas sample is in a different state at the end than it was at the beginning. The process cannot be done over unless the gas is recompressed by a work input. To be practical, a heat engine must operate in a cycle. The second law refers to a heat engine operating in a cycle, so this process is consistent with the second law of thermodynamics.

P22.59 The heat transfer over the paths CD and BA is zero since they are adiabatic.

Over path BC : $Q_{BC} = nC_p(T_C - T_B) > 0$

Over path DA : $Q_{DA} = nC_v(T_A - T_D) < 0$

Therefore, $|Q_c| = |Q_{DA}|$ and $Q_h = Q_{BC}$

The efficiency is then

$$e = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{(T_D - T_A)C_v}{(T_C - T_B)C_p}$$

$$e = 1 - \frac{1}{\gamma} \left[\frac{T_D - T_A}{T_C - T_B} \right]$$

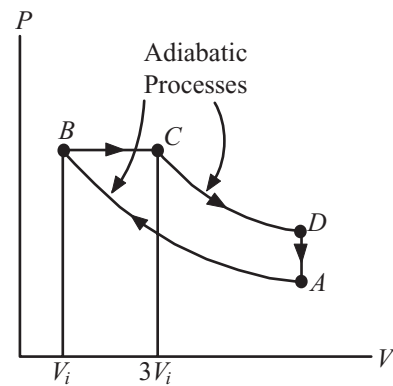


FIG. P22.59

P22.60 Simply evaluate the maximum (Carnot) efficiency.

$$e_c = \frac{\Delta T}{T_h} = \frac{4.00 \text{ K}}{277 \text{ K}} = 0.0144$$

The proposal does not merit serious consideration. Operating between these temperatures, this device could not attain so high an efficiency.

***P22.61** (a) 20.0°C

(b)
$$\Delta S = mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2} = 1.00 \text{ kg} (4.19 \text{ kJ/kg} \cdot \text{K}) \left[\ln \frac{T_f}{T_1} + \ln \frac{T_f}{T_2} \right]$$

$$= (4.19 \text{ kJ/K}) \ln \left(\frac{293}{283} \cdot \frac{293}{303} \right)$$

(c) $\Delta S =$ +4.88 J/K

(d) Yes, the mixing is irreversible. Entropy has increased.

P22.62 (a) Use the equation of state for an ideal gas

$$V = \frac{nRT}{P}$$

$$V_A = \frac{1.00(8.314)(600)}{25.0(1.013 \times 10^5)} = \boxed{1.97 \times 10^{-3} \text{ m}^3}$$

$$V_C = \frac{1.00(8.314)(400)}{1.013 \times 10^5} = \boxed{32.8 \times 10^{-3} \text{ m}^3}$$

Since AB is isothermal, $P_A V_A = P_B V_B$

and since BC is adiabatic, $P_B V_B^\gamma = P_C V_C^\gamma$

Combining these expressions,
$$V_B = \left[\left(\frac{P_C}{P_A} \right) \frac{V_C^\gamma}{V_A} \right]^{1/(\gamma-1)} = \left[\left(\frac{1.00}{25.0} \right) \frac{(32.8 \times 10^{-3} \text{ m}^3)^{1.40}}{1.97 \times 10^{-3} \text{ m}^3} \right]^{1/0.400}$$

$$V_B = \boxed{11.9 \times 10^{-3} \text{ m}^3}$$

Similarly,
$$V_D = \left[\left(\frac{P_A}{P_C} \right) \frac{V_A^\gamma}{V_C} \right]^{1/(\gamma-1)} = \left[\left(\frac{25.0}{1.00} \right) \frac{(1.97 \times 10^{-3} \text{ m}^3)^{1.40}}{32.8 \times 10^{-3} \text{ m}^3} \right]^{1/0.400}$$

or
$$V_D = \boxed{5.44 \times 10^{-3} \text{ m}^3}$$

Since AB is isothermal, $P_A V_A = P_B V_B$

and
$$P_B = P_A \left(\frac{V_A}{V_B} \right) = 25.0 \text{ atm} \left(\frac{1.97 \times 10^{-3} \text{ m}^3}{11.9 \times 10^{-3} \text{ m}^3} \right) = \boxed{4.14 \text{ atm}}$$

Also, CD is an isothermal and
$$P_D = P_C \left(\frac{V_C}{V_D} \right) = 1.00 \text{ atm} \left(\frac{32.8 \times 10^{-3} \text{ m}^3}{5.44 \times 10^{-3} \text{ m}^3} \right) = \boxed{6.03 \text{ atm}}$$

Solving part (c) before part (b):

(c) For this Carnot cycle,
$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{400 \text{ K}}{600 \text{ K}} = \boxed{0.333}$$

(b) Energy is added by heat to the gas during the process AB . For the isothermal process, $\Delta E_{\text{int}} = 0$.

and the first law gives
$$Q_{AB} = -W_{AB} = nRT_h \ln \left(\frac{V_B}{V_A} \right)$$

or
$$\begin{aligned} |Q_h| = Q_{AB} &= 1.00 \text{ mol} (8.314 \text{ J/mol} \cdot \text{K}) (600 \text{ K}) \ln \left(\frac{11.9}{1.97} \right) \\ &= 8.97 \text{ kJ} \end{aligned}$$

Then, from
$$e = \frac{W_{\text{eng}}}{|Q_h|}$$

the net work done per cycle is
$$W_{\text{eng}} = e_c |Q_h| = 0.333(8.97 \text{ kJ}) = \boxed{2.99 \text{ kJ}}$$

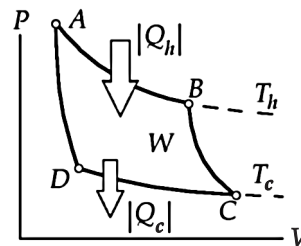


FIG. P22.62

ANSWERS TO EVEN PROBLEMS

P22.2 13.7°C

P22.4 (a) 29.4 L/h (b) 185 hp (c) 527 N·m (d) 1.91×10^5 W

P22.6 (a) 24.0 J (b) 144 J

P22.8 (a) 2.93 (b) coefficient of performance for a refrigerator (c) The cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

P22.10 (a) 870 MJ (b) 330 MJ

P22.12 (a) 0.300 (b) 1.40×10^{-3} (c) 2.00×10^{-3}

P22.14 33.0%

P22.16 (a) 5.12% (b) 5.27 TJ/h (c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

P22.18 (a) $|Q_c|/\Delta t = 700 \text{ kW}(T_h + 766 \text{ K})/(T_h - 383 \text{ K})$ The exhaust power decreases as the firebox temperature increases. (b) 1.87 MW (c) $3.84 \times 10^3 \text{ K}$ (d) No answer exists. The energy exhaust cannot be that small.

P22.20 (a)

State	P (kPa)	V (L)	T (K)
A	1400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b)

Process	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
A→B	6.58	-6.58	0
B→C	0	-4.98	-4.98
C→D	-5.02	5.02	0
D→A	0	4.98	4.98
ABCD	1.56	-1.56	0

(c) 23.7%; see the solution

P22.22 11.8

P22.24 1.17 J

P22.26 1.86

P22.28 (a), (b) see the solution (c) $Q_h = 149 \text{ J}$; $|Q_c| = 65.0 \text{ J}$; $W_{\text{eng}} = 84.3 \text{ J}$ (d) 56.5%
(e) $1.42 \times 10^3 \text{ rev/min}$

P22.30 -610 J/K

P22.32 (a) The process is isobaric because it takes place under constant atmospheric pressure. The heating process is not adiabatic (because energy goes in by heat), isothermal (T goes up), isovolumetric (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. (b) 402 kJ (c) 1.20 kJ/K

P22.34 3.27 J/K

P22.36 +1.18 J/K

P22.38 718 J/K

P22.40 0.507 J/K

P22.42 (a) 2 heads and 2 tails (b) All heads or all tails (c) 2 heads and 2 tails

P22.44 8.36 MJ/K

P22.46 (a) 39.4 J (b) 65.4 rad/s = 625 rev/min (c) 293 rad/s = 2 790 rev/min

P22.48 (a) 0.476 J/K (b) 417 J (c) $W_{\text{net}} = T_1 \Delta S_U = 167 \text{ J}$

P22.50 (a) 3.19 cal/K (b) 98.19°F, 2.59 cal/K This is significantly less than the estimate in part (a).

P22.52
$$\frac{\mathcal{P} T_c}{(T_h - T_c) c \Delta T}$$

P22.54 (a) 4.10 kJ (b) 14.2 kJ (c) 10.1 kJ (d) 28.8% (e) The three-process engine considered in this problem has much lower efficiency than the Carnot efficiency.

P22.56 $nC_p \ln 3$

P22.58 (a) see the solution (b) While it lasts, this process does convert all of the energy input into work output. But the gas sample is in a different state at the end than it was at the beginning. The process cannot be done over unless the gas is recompressed by a work input. To be practical, a heat engine must operate in a cycle. The second law refers to a heat engine operating in a cycle, so this process is consistent with the second law of thermodynamics.

P22.60 The proposal does not merit serious consideration. Operating between these temperatures, this device could not attain so high an efficiency.

P22.62 (a)

	P , atm	V , L
A	25.0	1.97
B	4.14	11.9
C	1.00	32.8
D	6.03	5.44

(b) 2.99kJ (c) 33.3%